1

(a) Use the formal definition of the derivative to find $f'(3)$ if $f(x) = \dfrac{1}{x-6}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{3+h-6} + \frac{1}{3}}{h} = \lim_{h \to 0} \frac{3 + (-3+h)}{(-3+h)3h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{(-3+h)3\cancel{h}} = \boxed{-\frac{1}{9}}$$

(b) Find the first and second derivatives of $g(x) = \ln(6 - x)$

$$g(x) = \ln(6-x)$$

$$g'(x) = \frac{1}{6-x} \cdot (-1) = \frac{1}{x-6}$$

$$g''(x) = -\frac{1}{(x-6)^2}$$

2. Find the derivative of the following functions.

(a) $y = \left(x^3 - x^{-3}\right)^{1/3}$

$$y' = \frac{1}{3}\left(x^3 - x^{-3}\right)^{-2/3} \cdot \left(3x^2 + 3x^{-4}\right)$$

(b) $y = 2^{\sqrt{x}} + 2^2$

$$y' = 2^{\sqrt{x}} \cdot \ln(2) \cdot \frac{1}{2} x^{-1/2}$$

(c) $y = \dfrac{e^{5x}}{1 + \cos(3x)}$

$$y' = \frac{(1 + \cos(3x)) \cdot 5e^{5x} - e^{5x} \cdot (-\sin(3x)) \cdot 3}{[1 + \cos(3x)]^2}$$

3 Find dy/dx for the following functions.

(a) $\cos(y^2) = y\sin(8x)$

$$-\sin(y^2)\cdot 2y\cdot y' = y'\sin(8x) + y\cdot 8\cos(8x)$$

$$y'\left[-2y\sin(y^2) - \sin(8x)\right] = 8y\cos(8x)$$

$$y' = -\frac{8y\cos(8x)}{2y\sin(y^2) + \sin(8x)}$$

(b) $y = x^{(x^2+x)}$

$$\ln(y) = (x^2+x)\ln(x)$$

$$\frac{1}{y}\cdot y' = (x^2+x)\cdot\frac{1}{x} + (2x+1)\cdot\ln(x)$$

$$= x+1 + (2x+1)\ln(x)$$

$$\Rightarrow y' = \left[x+1 + (2x+1)\ln(x)\right]x^{x^2+x}$$

4

(a) Write the equation for the linearization to $f(x) = \ln(3 - x)$ at $a = 1$

$L(x) = f(a) + f'(a)(x-a)$

$f'(x) = \frac{1}{3-x} \cdot (-1) = \frac{1}{x-3}$

$\Rightarrow L(x) = \ln(2) - \frac{1}{2}(x-1)$

(b) Use your work from (a) to estimate $\ln(1.9)$.

$\ln(1.9) = f(1.1) \approx L(1.1)$

$L(1.1) = \ln(2) - \frac{1}{2}(1.1-1) = \ln(2) - .05$

5 Cardiac output (the volume of blood pumped by the heart per minute) can be calculated with the formula

$$C = \frac{Q}{D},$$

where Q is the number of milliliters of CO_2 exhaled per minute and D is the difference between the CO_2 concentration in blood pumped to the lungs and blood returning from the lungs.

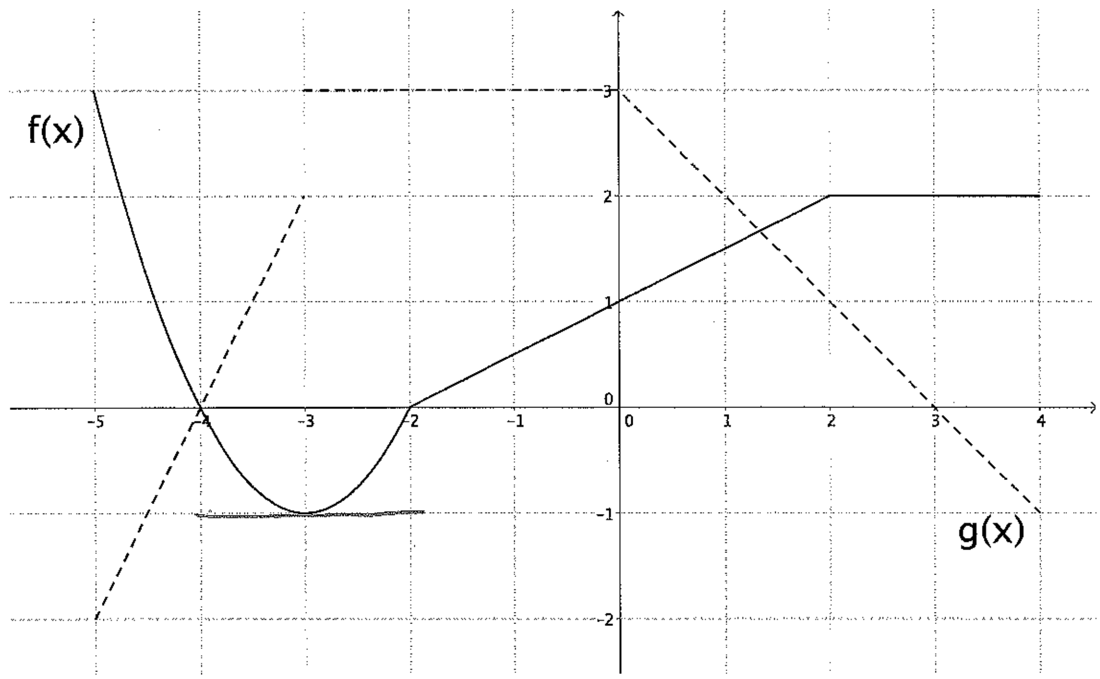
Suppose that when $Q = 233$ and $D = 41$, we also know that D is decreasing at the rate of 2 units a minute but that Q remains unchanged. How is cardiac output changing at this moment?

Given info: $\frac{dD}{dt} = -2 \ \frac{\text{units}}{\text{min}}$ $\quad \frac{dQ}{dt} = 0$ (i.e. Q is a constant)

$$\frac{dC}{dt} = Q \cdot \frac{-1}{D^2} \cdot \frac{dD}{dt}$$

$$\Rightarrow \frac{dC}{dt} = \frac{-233}{41^2} \cdot (-2) = \frac{466}{41^2}$$

6 Use the graph below to find the quantities listed. If the quantity does not exist, explain why.



(a) $f'(-3)$

tangent line is horizontal, thus $f'(-3) = 0$

(b) $g'(1)$

$= \dfrac{g(2)-g(0)}{2-0} = \dfrac{1-3}{2} = -1$

(c) $g'(3)$

Same as $g'(1)$

$= -1$

(d) $\frac{d}{dx}[f(g(2x))]$ at $x = 1$

$= f'(g(2x)) \cdot g'(2x) \cdot 2$ at $x=1$

$= f'(g(2)) \cdot g'(2) \cdot 2$

$= f'(1) \cdot g'(2) \cdot 2$

$= \frac{1}{2} \cdot (-1) \cdot 2 = -1$