MA 137: Calculus I for the Life Sciences

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Order of magnitude

Figure: A juvenile *Brookesia micra* on a finger tip, By Frank Glaw et al (2012) Rivaling the World's Smallest Reptiles: Discovery of Miniaturized and Microendemic New Species of Leaf Chameleons (Brookesia) from Northern Madagascar. PLoS ONE 7(2): e31314. doi:10.1371/journal.pone.0031314.

Topics covered during last class

- Exponential Growth and Decay.
- Doubling-time.
- Radioactive Decay.
- Half-life.

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- The Logarithmic Scale.
- Order of magnitude.
- Transformations into Linear Functions.
- Semilog Plots.

The Logarithmic Scale

Definition

A **logarithmic scale** is a nonlinear scale according to which multiples of 10 are equally distant. It is used when there is a large range of quantities. Common uses include

- lengths of organisms,
- earthquake strength,
- sound loudness,
- light intensity, and
- pH of solutions.

Section 1.3: Order of magnitude

Definition

The order of magnitude of a number is the smallest power of 10 required to represent that number.

- If two quantities differ by a factor of 10, they differ by one order of magnitude,
- If two quantities differ by a factor of 100, they differ by two orders of magnitude, and
- so on.

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Example (Order of magnitude)

1. A 1.8 m tall human and a 25 m long blue whale differ by about one order of magnitude.

Section 1.3: Order of magnitude

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Answer: $y = 3$.

The *pH* Scale

Chemists measured the acidity of a solution by giving its hydrogen ion concentration until Sorensen, in 1909, de ned a more convenient measure:

$$pH = -\log[H^+]$$

where $[H^+]$ is the concentration of hydrogen ions measured in moles per liter (M).

Solutions are defined in terms of the pH as follows:

- those with $pH = 7$ (or $[H^+] = 10^{-7}M$) are neutral,
- those with $pH < 7$ (or $[H^+] > 10^{-7}M$) are acidic,
- those with $pH > 7$ (or $[H^+] < 10^{-7}M$) are basic.

The *pH* Scale

Example (Finding *pH*)

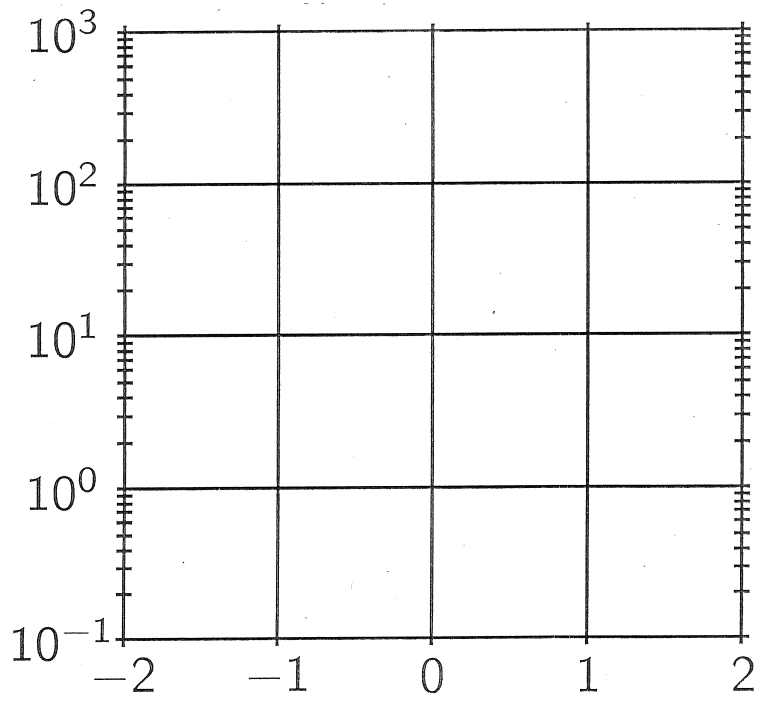
The hydrogen ion concentration of a sample of each substance is given. Calculate the *pH* of the substance.

- Lemon juice: $[H^+] = 5.0 \times 10^{-3} M$),
- Seawater: $[H^+] = 5.0 \times 10^{-9} M$).

Semilog Plots

In biology its common to use a semilog plot to see whether data. points are appropriately modeled by an exponential function.

- This means that instead of plotting the points (x, y), we plot the points $(x, \log y)$.
- In other words, we use a logarithmic scale on the vertical axis.



Section 1.3: Transformations into Linear Functions

Recall the definition of an exponential function.

Definition

The function f is an exponential function with base a if

$$f(x) = a^x$$

where a is a positive constant other than 1.

Example

Graph the following functions

- $f(x) = 2.5 \cdot 3^x$.
  `http://sagecell.sagemath.org/?q=hlkegn`.
- $f(x) = 0.3979 + 0.4771x$.
  `http://sagecell.sagemath.org/?q=cwpogl`.

Rule (Semilog Plots)

If we start with an exponential function of the form $y = a \cdot b^x$ *and take logarithms on both sides,*

$$\log y = \log a + x \log b$$

Let $Y = \log y$, $m = \log b$, *and* $c = \log a$. *Then*

$$Y = mx + c$$

This is the equation of a line with slope m *and* y*-intercept* c.

So if we obtain experimental data that we suspect might possibly be exponential, then we could graph a semilog scatter plot and see if it is approximately linear.