MA 138 – Calculus 2 with Life Science Applications

Course Introduction & Section 6.3 (Applications of integration)

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Instructor

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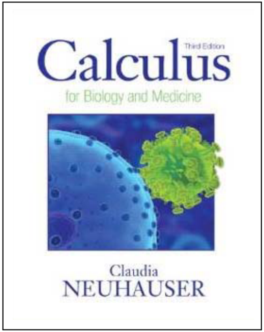
003 TR 12:00-12:50pm – CB 339

Textbook



Title: Calculus for Biology and Medicine

Author: Claudia Neuhauser

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Course Outline for MA 138

Ch. 6: Applications of integration

Ch. 7: Integration techniques and computational methods

Ch. 8: Differential equations

Ch. 9: Linear algebra and analytic geometry

Ch. 10: Multivariable calculus

Ch. 11: Systems of differential equations

Grading

You will be able to obtain a **maximum of 500 points** in this class, divided as follows:

- Three 2-hour exams, 100 points each;
- Final exam, 100 points;
- Homework, 50 points;
- Weekly quizzes, 50 points.

Your final grade for the course will be based on the total points you have earned as follows:

A: 450-500	B: 400-449	C: 350-399	D: 300-349	E: 0-299
$\geq 90\%$	$\geq 80\%$	$\geq 70\%$	$\geq 60\%$	$< 60\%$

Exams (Regular and Alternate)

Regular Exams will be given on

- Tuesday, February 5 — 5:00-7:00pm
- Tuesday, March 5 — 5:00-7:00pm
- Tuesday, April 9 — 5:00-7:00pm
- Wednesday, May 1 — 8:30-10:30pm

Alternate Exams for Exams 1-3 are given on the same days as the regular exams from 7:30-10:00pm (February 5, March 5, April 9).

Review Sessions for exams 1-3 will be held on Monday February 4, March 4 and April 8 from 5:00-7:00pm.

Homework

- The homework has **two components**: **online** and **handwritten**. Each will count as half of the final homework grade. The online problems cover the more routine aspects of the class. The written homework problems are usually more conceptual and are often motivated by problems from the Life Sciences.
- The online homework (WeBWorK) can be accessed through

  https://webwork.as.uky.edu/webwork2/MA138S19/
- Your username is your **Link Blue user ID** (use capital letters!) and your password is **your 8 digit student ID number**.
- You can try online problems as many times as you like.
  The system will tell you if your answer is correct or not.
  You can email the TA a question from each of the problem. TAs will always do their best to respond within 24 hours.
- **Don't wait until the last minute!**

¿Minoring in Mathematics?

To obtain a **minor in Mathematics**, a student who has completed MA 137/138 Calculus I and II must complete the following:

1. MA 213 – Calculus III (4 credits)
2. MA 322 – Matrix Algebra and Its Applications (3 credits)
3. Six additional credit hours of Mathematics courses (=two courses) numbered greater than 213. Possible courses include: MA 214, MA 261, MA 320, MA 321, **MA 327 (Introduction to game theory)**, MA 330, MA 341, MA 351, MA 361, or any 400 level math course
4. We also just established a new cross-listed course at the upper level in Mathematics:

   MA 337/BIO 337: Mathematical Modeling in the Life Sciences

Thus you need 13 additional credit hours in Mathematics classes.

Course Outline for MA 337/BIO 337 (Spring 2019):

Biology presents complex problems requiring quantitative approaches to tackle them.

The first half of the course will focus on discrete models in mathematical biology, such as difference equations and matrix population models.

The second half of the term will focus on continuous models, namely differential equations in mathematical biology. Examples for each type of modeling may be taken from ecology, epidemiology, molecular networks, or gene regulatory networks. By the end of the term, you will have experience posing biological problems and using mathematics to elucidate them.

Course topics will be structured as follows:

Discrete Models

- Single-species discrete population models, their equilibria and stability. Examples include: Simplified Logistic model, Ricker Model, Beverton-Holt model.
- Matrix population models. Examples will be taken from ecology.
- Analysis of multidimensional discrete models. Application: Discrete Epidemic Models.

Differential Equations

- One-dimensional continuous population models and their qualitative analysis.
- Introduction to Epidemiological Models: the Kermack-McKendrick SIR and SIS models with no demography.
- Properties of planar systems, their equilibria, phase plane analysis, linear stability analysis, Poincare-Bendixson Trichotomy and Dulac-Bendixson Criterion. Some applications: Lotka-Volterra predator-prey model, SIR model with demography
- Linear stability analysis for higher order systems. Application: deriving the basic reproduction number R_0 in complex epidemiological models.
- Bistability in epidemiological models.

Section 6.3: Applications of Integration

We are interested in the following three applications of integrals:

(1) **average** of a continuous function on $[a, b]$;

(2) **area between curves**;

(3) **cumulative change**.

Average Values

It is easy to calculate the average value of finitely many numbers $y_1, y_2, \ldots, y_n$:

$$y_{\text{avg}} = \frac{y_1 + y_2 + \cdots + y_n}{n}$$

But how do we compute the average temperature during a day if infinitely many temperature readings are possible?

In general, let's try to compute the average value of a function $y = f(x)$, $a \leq x \leq b$. We start by dividing the interval $[a, b]$ into n equal subintervals, each with length $\Delta x = (b - a)/n$. Then we choose points $c_1, \ldots, c_n$ in successive subintervals and calculate the average of the numbers $f(c_1), \ldots, f(c_n)$:

$$\frac{f(c_1) + \cdots + f(c_n)}{n}$$

Since $\Delta x = (b-a)/n$, we can write $1/n = \Delta x/(b-a)$ and the average value becomes

$$\frac{f(c_1)\Delta x + \cdots + f(c_n)\Delta x}{b-a} = \frac{1}{b-a}\sum_{i=1}^{n} f(c_i)\Delta x.$$

If we let n increase, we would be computing the average value of a large number of closely spaced values. More precisely,

$$\lim_{n\to\infty} \frac{1}{b-a}\sum_{i=1}^{n} f(c_i)\Delta x = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Average of a Continuous Function on $[a, b]$

Assume that $f(x)$ is a continuous function on $[a, b]$. The average value of f on the interval $[a, b]$ is defined to be

$$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx,$$

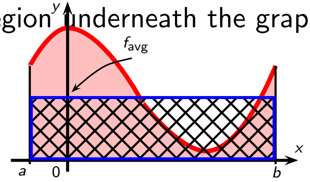
Geometric Meaning

Mean Value Theorem for Definite Integrals

Assume that $f(x)$ is a continuous function on $[a, b]$. Then there exists a number $c \in [a, b]$ such that

$$f(c)(b-a) = \int_a^b f(x)\,dx.$$

That is, when f is continuous, there exists a number c such that $f(c) = f_{\text{avg}}$. If f is a continuous, positive valued function, f_{avg} is that number such that the rectangle with base $[a, b]$ and height f_{avg} has the same area as the region underneath the graph of f from a to b.



Example 1 (Online Homework #14)

If a cup of coffee has temperature 95°C in a room where the temperature is 20°C, then, according to Newton's Law of Cooling, the temperature of the coffee after t minutes is

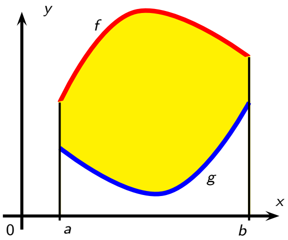
$$T(t) = 20 + 75e^{-t/50}.$$

What is the average temperature (in degrees Celsius) of the coffee during the first half hour?

Area Between Curves

Assume f and g are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$. The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the linesx $= a$, $x = b$, is

$$A = \int_a^b [f(x) - g(x)]\, dx.$$



Example 2 (Online Homework #2)

Find the area of the region enclosed by the two functions $y = 7x^2$ and $y = x^2 + 6$.

Example 3 (Online Homework #3)

Find the area between $y = 8\sin x$ and $y = 10\cos x$ over the interval $[0, \pi]$. Sketch the curves if necessary.

Example 4 (Online Homework #4)

Find the area between $y = e^x$ and $y = e^{4x}$ over $[0, 1]$.

Example 5 (Online Homework #6)

Find the area of the quadrangle with vertices $(4,2)$, $(-5,4)$, $(-2,-4)$, and $(3,-3)$.

Example 6 (Online Homework #7)

Consider the area between the graphs $x + y = 14$ and $x + 6 = y^2$.

This area can be computed in two different ways using integrals.

- First of all it can be computed as a sum of two integrals

$$\int_a^b f(x)\,dx + \int_b^c g(x)\,dx$$

where $a =$ ____, $b =$ ____, $c =$ ____, and $f(x) =$ ____ $g(x) =$ ____.

- Alternatively this area can be computed as a single integral

$$\int_\alpha^\beta h(y)\,dy$$

where $\alpha =$ ____, $\beta =$ ____, and $h(y) =$ ____.

Example 7 (Online Homework #5)

Find the value(s) of c such that the area of the region bounded by the parabolae $y = x^2 - c^2$ and $y = c^2 - x^2$ is 1944.

Cumulative Change

Suppose that we have a population whose size at time t is given by $N(t)$. Suppose further that its rate of growth is given by the initial value problem

IVP: $$\frac{dN}{dt} = f(t) \qquad N(0) = N_0.$$

Then, by Part I of the Fundamental Theorem of Calculus we have that

$$N(t) = \int_0^t f(u)\,du + C$$

represents all antiderivatives of $f(t)$ [or dN/dt].

Now, $N(0) = \underbrace{\int_0^0 f(u)\,du}_{=0} + C = C$ so $C = N_0 = N(0)$. Therefore

$$N(t) = \int_0^t f(u)\,du + N_0 \qquad \text{or} \qquad \boxed{N(t) - N(0) = \int_0^t f(u)\,du}.$$

More generally, the IVP: $\dfrac{dN}{dt} = f(t) \qquad N(a) = N_a$ has solution

$$\boxed{N(t) - N(a) = \int_a^t f(u)\,du = \int_a^t \frac{dN}{du}\,du}.$$

That is

$$\left\{ \begin{array}{c} \text{cumulative change} \\ \text{on the interval } [a, t] \end{array} \right\} = \int_a^t \left\{ \begin{array}{c} \text{instantaneous rate of} \\ \text{change at time } u \end{array} \right\} du$$

Similarly, if $p(t)$ is the position function of an object at time t, then

$$\frac{dp}{dt} = v(t) \qquad\qquad p(a) = p_a$$

gives $\rightsquigarrow$

$$\boxed{\underbrace{p(b) - p(a)}_{\text{distance traveled on } [a,b]} = \int_a^b v(t)\,dt = \int_a^b \frac{dp}{dt}\,dt}.$$

Example 8 (Problem #18, Section 6.3, page 321)

Suppose the change in biomass $B(t)$ at time t during the interval $[0, 12]$ follows the equation

$$\frac{dB}{dt} = \cos\left(\frac{\pi}{6}t\right).$$

How does the biomass at time $t = 12$ compare to the biomass at time $t = 0$?

Example 9 (Problem #22, Section 6.3, page 322)

If $\frac{dw}{dx}$ represents the rate of change of the weight of an organism of age x, explain what

$$\int_3^5 \frac{dw}{dx}\,dx$$

means.