MA 138 – Calculus 2 with Life Science Applications
Integration by Parts
(Section 7.2)

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Section 7.2: Integration by Parts

We saw that **integration by parts is the product rule in integral form.**

We also recall the following **general formula**:

Rule for Integration by Parts

If $f(x)$ and $g(x)$ are differentiable functions, then

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx;$$

$$\int_a^b f(x)g'(x)\,dx = f(x)g(x)\Big]_a^b - \int_a^b f'(x)g(x)\,dx.$$

Example 1

A particle that moves along a straight line has velocity

$$v(t) = t^2 e^{-2t}$$

meters per second after t seconds.

How many meters will it travel during the first t seconds?

Example 2 (Online Homework # 9)

Suppose that $f(1) = 4$, $f(4) = 6$, $f'(1) = -5$, $f'(4) = -5$, and f'' is continuous. Find the value of

$$\int_1^4 x f''(x)\, dx.$$

Example 3 (Problem # 8, Section 7.2, page 372)

Evaluate the indefinite integral: $\int 3xe^{-x/2}\,dx.$

Example 4 (Online Homework # 6)

Find the integral: $\int_0^1 x^2 \sqrt[4]{e^x}\, dx$.

Example 5 (Problem # 35, Section 7.2, page 373)

Evaluate the indefinite integral: $\displaystyle\int \frac{1}{x} \ln x \, dx.$

Example 6 (Problem # 48, Section 7.2, page 373)

Evaluate the definite integral: $\int_0^1 x^3 \ln(x^2+1)\,dx.$

Useful aside: Trigonometric addition formulas

- We also used the double angle formulas

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha \quad \text{and} \quad \sin(2\alpha) = 2\sin\alpha\cos\alpha$$

  to compute $\int \cos^2 x\,dx$ and $\int \sin x \cos x\,dx$.

- Is there a 'simple' way to remember formulas of this kind?

- **Euler's formula** establishes the fundamental relationship between the trigonometric functions and the complex exponential function. It states that, for any real number x,

$$e^{ix} = \cos x + i\sin x,$$

  where i is the imaginary unit ($i^2 = -1$).

- For any α and β, using Euler's formula, we have

$$\begin{aligned}\cos(\alpha+\beta)+i\sin(\alpha+\beta) &= e^{i(\alpha+\beta)}\\ &= e^{i\alpha}\cdot e^{i\beta}\\ &= (\cos\alpha+i\sin\alpha)\cdot(\cos\beta+i\sin\beta)\\ &= (\cos\alpha\cos\beta+i^2\sin\alpha\sin\beta)\\ &\qquad +i(\sin\alpha\cos\beta+\cos\alpha\sin\beta).\end{aligned}$$

- Thus, by comparing the terms, we obtain

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta.$$

- Thus, by setting $\alpha=\beta$, we obtain

$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha \quad \text{and} \quad \sin(2\alpha)=2\sin\alpha\cos\alpha.$$