WORKSHEET #19

#1

$$\begin{bmatrix} 1 & 0 \\ -7 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -7+6 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

so the eigenvalue is $\lambda = 1$

$$\begin{bmatrix} 1 & 0 \\ -7 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} = -6 \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

so the eigenvalue is $\lambda = -6$

Notice that the eigenvalues are the diagonal entries of the given matrix $\begin{bmatrix} 1 & 0 \\ -7 & -6 \end{bmatrix}$ as it has a zero in the upper corner.

#2

$$\begin{bmatrix} -7 & 2 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -5 \begin{bmatrix} x \\ y \end{bmatrix}$$

this corresponds to the 2 equations

$$\begin{cases} -7x + 2y = -5x \\ -3x - 2y = -5y \end{cases} \iff \begin{cases} -2x + 2y = 0 \\ -3x + 3y = 0 \end{cases}$$

Both equations simplify into

$$y = x$$

so we can pick as an eigenvector

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{as } \underline{y = x}$$

<u>Next</u>, consider $\lambda = -4$. We need

$$\begin{bmatrix} -7 & 2 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -4\begin{bmatrix} x \\ y \end{bmatrix}$$

which corresponds to

$$\begin{cases} -7x + 2y = -4x \\ -3x - 2y = -4y \end{cases} \iff \begin{cases} -3x + 2y = 0 \\ -3x + 2y = 0 \end{cases}$$

Hence $y = \frac{3}{2}x$ and we

can choose as eigenvector $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\boxed{\#3}$ We need to solve the equation

$$\det \begin{bmatrix} 22-\lambda & -72 \\ 6 & -20-\lambda \end{bmatrix} = 0$$

or $(22-\lambda)(-20-\lambda) + 6\cdot 72 = 0$

$$\iff \lambda^2 - 22\lambda + 20\lambda - 440 + 432 = 0$$

$$\iff \lambda^2 - 2\lambda - 8 = 0$$

trace of A $= +22-20 = 2$

determinant of A $22(-20) - 6(-72) = -8$

The equation factors as

$$(\lambda-4)(\lambda+2)=0$$

so $\lambda_1 = 4$ and $\lambda_2 = -2$

To find the eigenvectors proceed as in $\boxed{\#2}$.

#4

We know that for a 2x2 matrix the product of the 2 eigenvalues is the determinant of the matrix

So $\det \begin{bmatrix} -5 & -9 \\ -8 & k \end{bmatrix} = \lambda_1 \lambda_2 = 0$

$\lambda_2 = 0$

$$\Rightarrow \quad -5k - (-9)(-8) = 0$$

$$-5k - 72 = 0$$

$$k = -72/5$$

#5

$$\det \begin{bmatrix} 4-\lambda & k \\ -3 & -4-\lambda \end{bmatrix} = 0$$

$$\Leftrightarrow \quad (4-\lambda)(-4-\lambda) + 3k = 0$$

$$\lambda^2 - 16 + 3k = 0$$

$$\lambda^2 = 16 - 3k$$

or $\lambda = \pm\sqrt{16-3k}$

We need $16 - 3k > 0$ or $k < \frac{16}{3}$

#6

$$\begin{bmatrix} 2 & -6 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} = \underbrace{2}_{\text{eigenvalue } 2}\begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -6 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4-6 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix} = \underbrace{-}_{\text{eigenvalue } -1}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

<u>Now</u> we need to find c_1 and c_2 constants such that

$$\begin{bmatrix} 11 \\ 4 \end{bmatrix} = c_1\begin{bmatrix} -1 \\ 0 \end{bmatrix} + c_2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

or

$$\begin{cases} 11 = -c_1 + 2c_2 \\ 4 = \underbrace{\cancel{c_1 \cdot 0}}_{0} + c_2 \end{cases}$$

so $c_2 = 4$ and $11 = -c_1 + 2(4)$

$c_1 = 8 - 11$ $\quad c_1 = -3$

Thus $\boxed{\begin{bmatrix} 11 \\ 4 \end{bmatrix} = -3\begin{bmatrix} -1 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 2 \\ 1 \end{bmatrix}}$

Hence

$$A^{10} \cdot \begin{bmatrix} 11 \\ 4 \end{bmatrix} = A^{10}\left(-3\begin{bmatrix} -1 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$$

Using the properties of matrix multiplication

$$= -3\, A^{10}\begin{bmatrix} -1 \\ 0 \end{bmatrix} + 4\, A^{10}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

because they are eigenvectors

$$= -3 \cdot 2^{10} \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix} + 4\,(-1)^{10} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 2^{10} \\ 0 \end{bmatrix} + \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \cdot 2^{10} + 8 \\ 4 \end{bmatrix}$$