WORKSHEET #22

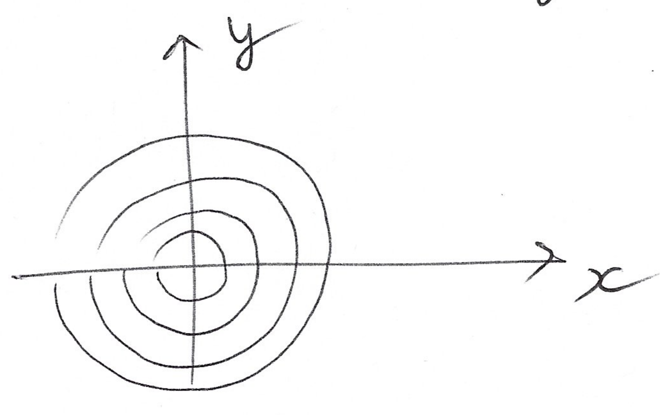
#1

(a) $f_1(x,y) = x^2 + y^2$

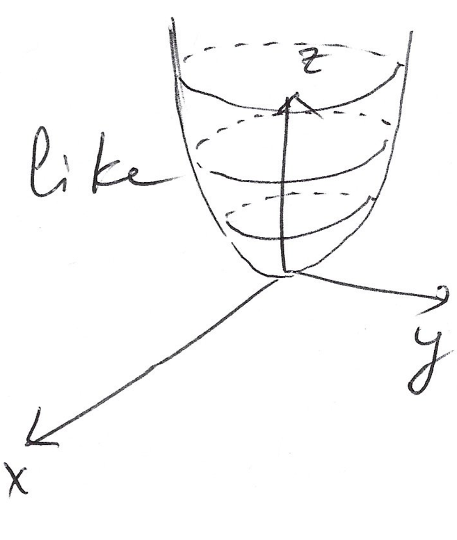
The level curves of $f_1(x,y)$ are obtained by setting $f_1(x,y) = c$

or $x^2 + y^2 = c$

These equations make sense for $c \geq 0$ and they represents concentric circles of radius $\sqrt{c}$



The graph in 3D looks like



To intersect with the x-z plane we need to set $y = 0$

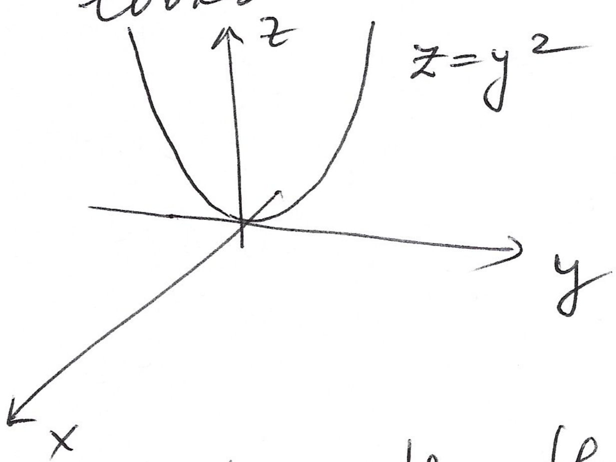
$$f_1(x, 0) = x^2 + 0^2 = x^2$$

So the graph is

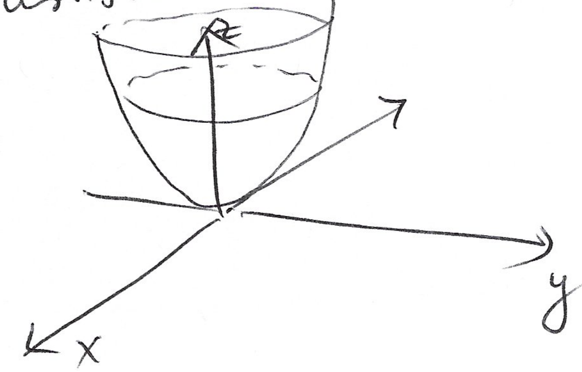
To intersect with the y-z plane we need to set $x = 0$

$$f_1(0, y) = 0^2 + y^2 = y^2$$

So the graph looks like



All this is consistent with the 3D graph

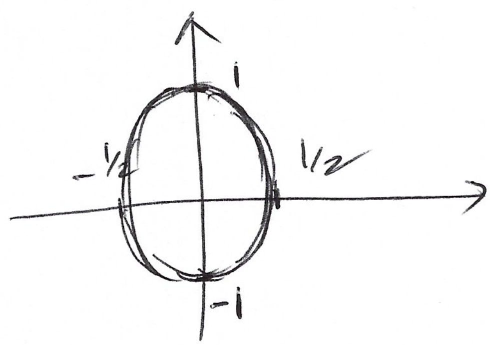


(b) $f_4(x,y) = 4x^2 + y^2$

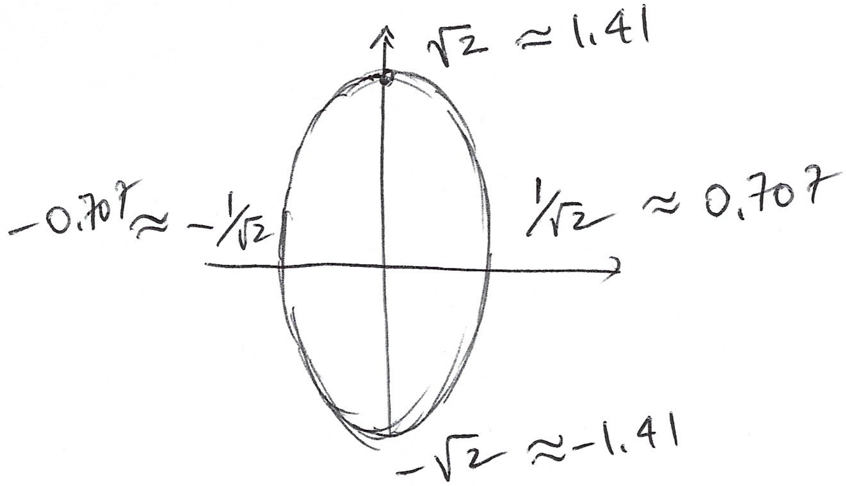
The level curves are of the form $4x^2+y^2=c$. These are ellipses and have solutions only for $c \geq 0$.

For $c=0$ $4x^2+y^2=0$ the only point satisfying the equation is $(0,0)$.

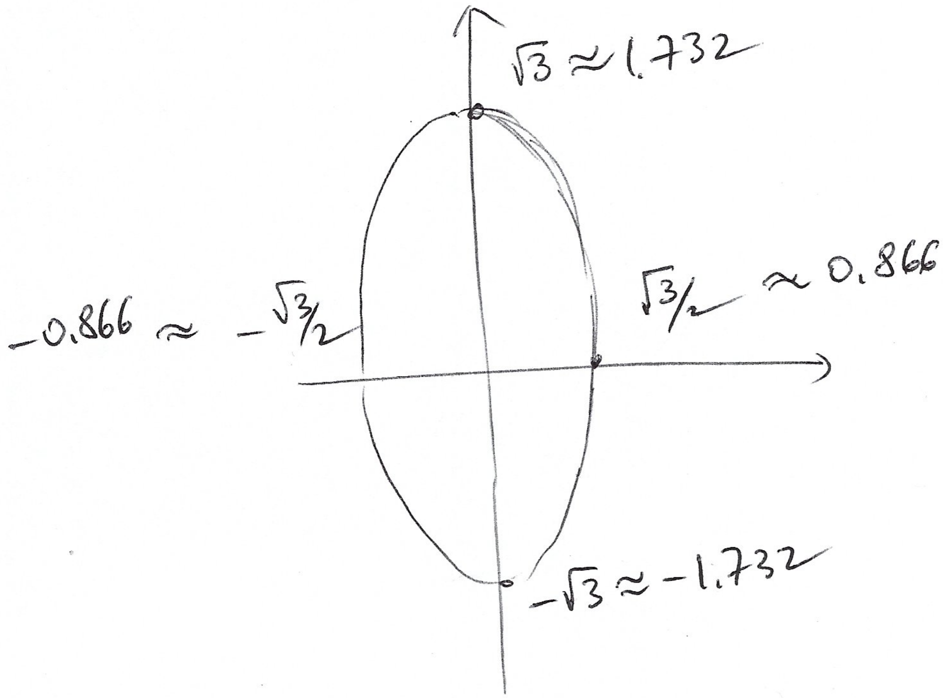
For $c=1$ $4x^2+y^2=1$ looks like



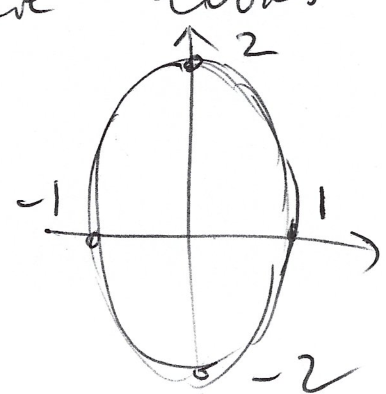
For $c=2$ $4x^2+y^2=2$ the graph of the level curve looks like



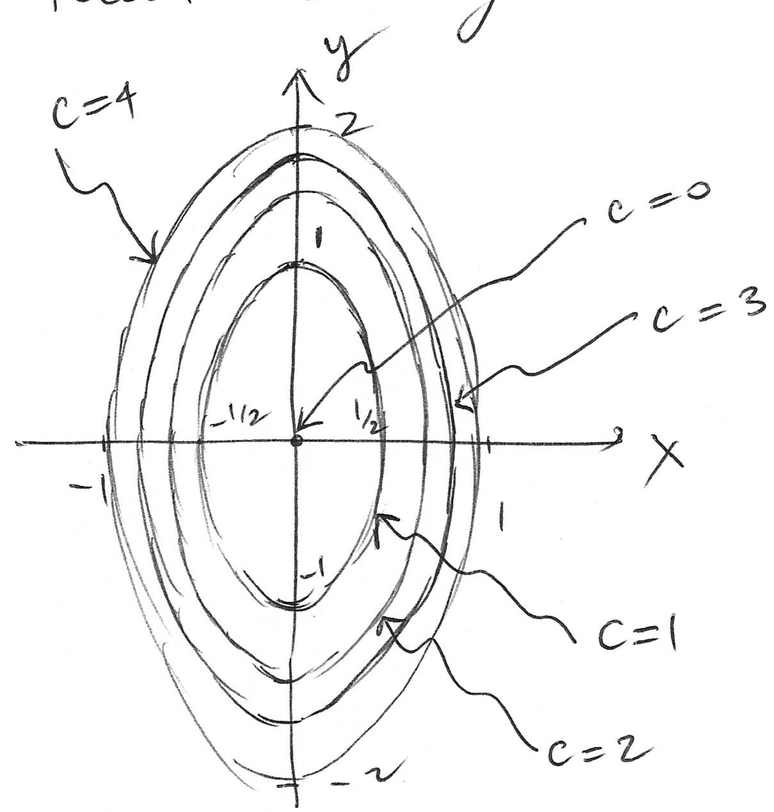
For $c = 3$ $\quad 4x^2 + y^2 = 3$ the graph of the level curve looks like



For $c = 4$ $\quad 4x^2 + y^2 = 4$ the graph of the level curve looks like



Plotting them all together



The intersection of $f_4(x,y)$ with the x-z plane is obtained by setting $y=0$

$$f_4(x,0) = 4x^2 + 0^2 = 4x^2$$

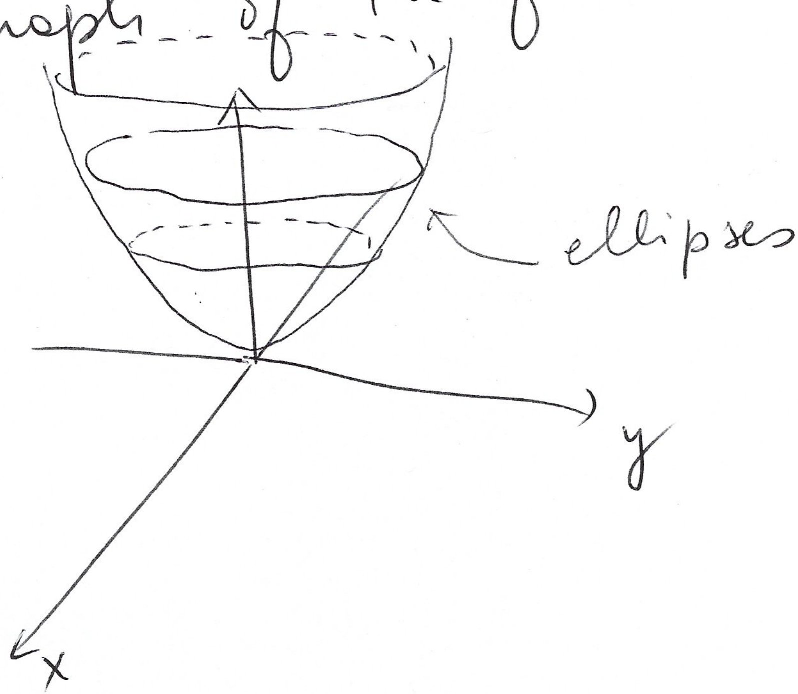
it is a parabola which is stretched by a factor of 4.

The intersection with the y-z plane is obtained by setting $x=0$

$$f_4(0,y) = 4 \cdot 0^2 + y^2 = y^2$$

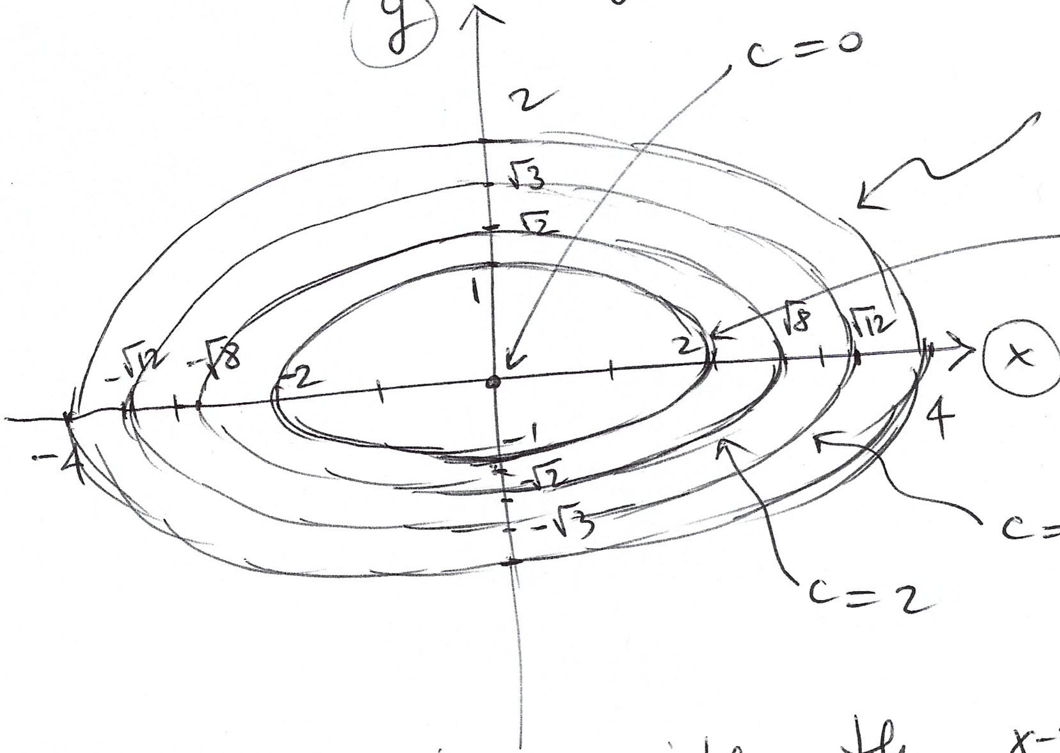
so that the graph is a parabola in the y-z plane

The graph of the function is



(c) for $a = 1/4$ $\quad f_{1/4}(x,y) = \frac{1}{4}x^2 + y^2$

The level curves for $c = 0, 1, 2, 3, 4$ are

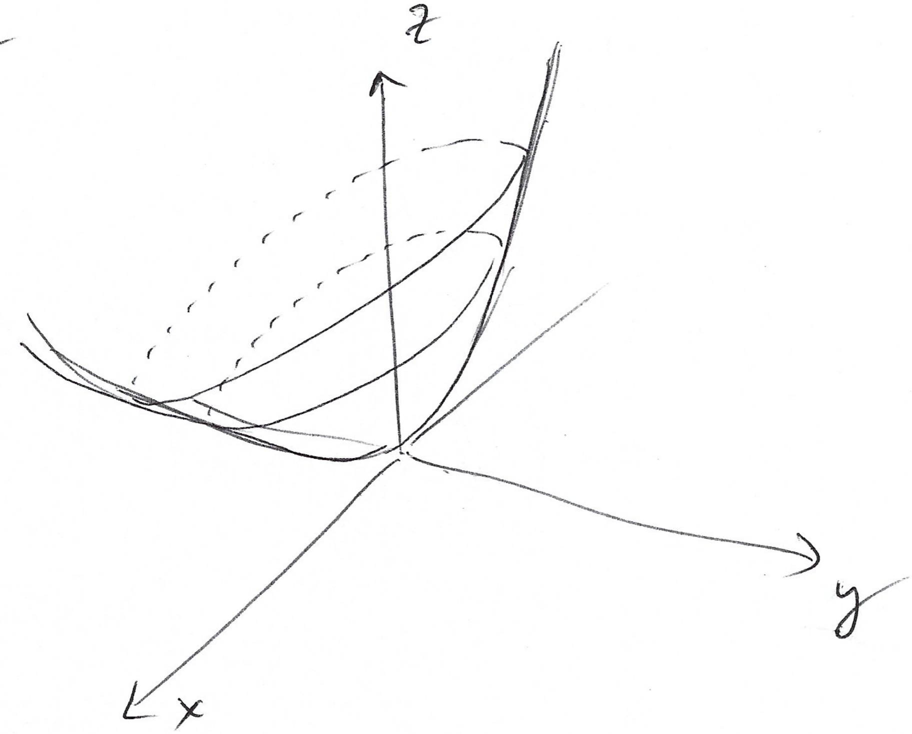


The intersection with the x-z plane is $f_{1/4}(x,0) = \frac{1}{4}x^2$

the intersection with the y-z plane

is $f_{1/4}(0,y) = y^2$

the overall graph of $f_{1/4}(x,y)$ looks like



For $a > 1$ it looks like the graph is squeezed toward the z-axis in the x-direction

Whereas for $a < 1$ it looks like the graph is stretched toward the x-axis in the x-direction.

$\boxed{\#2}$

$$\lim_{(x,y)\to(45,9)} x\cdot y\cdot\cos(x-5y) =$$

the properties of limits work also for functions of 2 variables -

Essentially we "evaluate" the function at that point

$$= 45\cdot 9\cdot\cos(45-5\cdot 9)$$

$$= 45\cdot 9\cdot\underbrace{\cos(0)}_{1} = 45\cdot 9 = 405$$

unless something "illegal" happens like in the next example

$\boxed{\#3}$

$$\lim_{(x,y)\to(0,0)} \frac{(9x+y)^2}{81x^2+y^2} =$$

$= $ using substitution $= \dfrac{0}{0}$

This is undefined so we require more investigation

(a)
$$\lim_{(x,y)\to(0,0)} \frac{(9x+y)^2}{81x^2+y^2} = \lim_{\substack{x\to 0\\ y=0}} \frac{(9x)^2}{81x^2}$$

along the x-axis

$$= \lim_{x\to 0} \frac{\cancel{81x^2}}{\cancel{81x^2}} = \lim_{x\to 0} 1 = 1$$

(b)
$$\lim_{(x,y)\to(0,0)} \frac{(9x+y)^2}{81x^2+y^2} = \lim_{\substack{y\to 0\\ x=0}} \frac{y^2}{y^2}$$

along the y-axis

$$= \lim_{y\to 0} 1 = 1$$

(c)
$$\lim_{(x,y)\to(0,0)} \frac{(9x+y)^2}{81x^2+y^2} = \lim_{\substack{x\to 0\\ y=x}} \frac{(9x+x)^2}{81x^2+x^2}$$

along $y=x$ line

$$= \lim_{\substack{x \to 0 \\ \text{as } y=x}} \frac{100x^2}{82x^2} = \lim_{x \to 0} \frac{100}{82} = \frac{100}{82} \approx 1.2195$$

(d) $$\lim_{\substack{(x,y) \to (0,0) \\ \text{along } y=mx}} \frac{(9x+y)^2}{81x^2+y^2} = \lim_{\substack{x \to 0 \\ \text{as } y=mx}} \frac{(9x+mx)^2}{81x^2+(mx)^2}$$

$$= \lim_{\substack{x \to 0 \\ \text{as } y=mx}} \frac{(9+m)^2 \cdot \cancel{x^2}}{(81+m^2)\cancel{x^2}} = \frac{(9+m)^2}{81+m^2}$$

the answer depends on the slope m according to the line we are approaching $(0,0)$

(e) hence the limit does not exist