WORKSHEET #23

#1 $$\lim_{(x,y)\to(0,0)} \frac{5x^2-y^2}{x^2+y^2}$$

(a) along $y=3x$

$$\lim_{\substack{x\to 0\\ y=3x}} \frac{5x^2-(3x)^2}{x^2+(3x)^2} = \lim_{x\to 0} \frac{5x^2-9x^2}{x^2+9x^2} =$$

$$= \lim_{x\to 0} \frac{-4x^2}{10x^2} = \underline{-0.4}$$

(b) along $y=4x$

$$\lim_{\substack{x\to 0\\ y=4x}} \frac{5x^2-(4x)^2}{x^2+(4x)^2} = \lim_{x\to 0} \frac{5x^2-16x^2}{x^2+16x^2}$$

$$= \lim_{x\to 0} \frac{-11x^2}{17x^2} = -{}^{11}/_{17} \approx -0.647$$

(c) along $y=mx$

$$\lim_{\substack{x\to 0\\ y=mx}} \frac{5x^2-(mx)^2}{x^2+(mx)^2} = \lim_{x\to 0} \frac{5x^2-m^2x^2}{x^2+m^2x^2}$$

$$= \lim_{x\to 0} \frac{5-m^2}{(1+m^2)}\cdot\frac{x^2}{x^2} = \frac{5-m^2}{1+m^2}$$

(d) Hence $\lim_{(x,y)\to(0,0)} \frac{5x^2-y^2}{x^2+y^2}$

does not exist because when we approach (0,0) through lines $y = mx$ the limit depends on "m".

$\boxed{\#2}$ $\lim_{(x,y)\to(2,3)} e^{\sqrt{5x^2+4y^2}} =$

(a)

$$= e^{\sqrt{\lim_{(x,y)\to(2,3)} (5x^2+4y^2)}} =$$

$$= e^{\sqrt{5(2)^2+4(3)^2}} \approx e^{\sqrt{56}} \approx 1778.1251$$

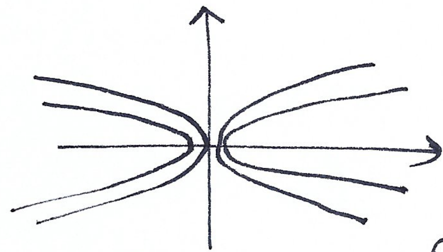
because lim behaves well with continuous functions and polynomials

(b) $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4} = \frac{0.0^2}{0^2+0^4} = \frac{0}{0}$

We need to investigate further the limit.

If we compute the limit along

$x = my^2$



these curves through $(0,0)$ we observe that

$$\lim_{\substack{(x,y)\to(0,0) \\ \text{along } x=my^2}} \frac{xy^2}{x^2+y^4} = \lim_{\substack{y\to 0 \\ \text{as } x=my^2\to 0}} \frac{(my^2)y^2}{(my^2)^2+y^4}$$

$$= \lim_{y\to 0} \frac{m\cancel{y^4}}{m^2\cancel{y^4}+\cancel{y^4}} = \lim_{y\to 0} \frac{m}{m^2+1} = \frac{m}{m^2+1}$$

the limit depends on m

Hence the limit does not exist

$\boxed{\#3}$ $$f(x,y) = \frac{7x+3y}{5x-9y}$$

- $$\frac{\partial f}{\partial x} = \frac{7(5x-9y)-(7x+3y)(5)}{(5x-9y)^2} =$$

$$= \frac{\cancel{35x} - 63y - \cancel{35x} - 15y}{(5x-9y)^2}$$

$$= \frac{-78y}{(5x-9y)^2}$$

- $$\frac{\partial f}{\partial y} = \frac{3(5x-9y)-(7x+3y)(-9)}{(5x-9y)^2}$$

$$= \frac{15x - \cancel{27y} + 63x + \cancel{27y}}{(5x-9y)^2}$$

$$= \frac{78x}{(5x-9y)^2}$$

$\boxed{\#4}$ $$f(x,y) = 6x^3y + 9xy^3$$

$$\frac{\partial f}{\partial x} = 18x^2y + 9y^3$$

$$\frac{\partial^2 f}{\partial x^2} = 36xy \qquad \frac{\partial^2 f}{\partial y \partial x} = 18x^2 + 27y^2$$

$=$ ✓

$$\frac{\partial f}{\partial y} = 6x^3 + 27xy^2$$

$$\frac{\partial^2 f}{\partial y^2} = 54xy \qquad \frac{\partial^2 f}{\partial x \partial y} = 18x^2 + 27y^2$$

<u>Note</u> $\quad \dfrac{\partial^2 f}{\partial y \partial x} = \dfrac{\partial^2 f}{\partial x \partial y}$ ✓