WORKSHEET #3

#1 $$\int \frac{3x-3}{(3x^2-6x+4)^5}\,dx$$

observe that the numerator is "essentially" the derivative of the trinomial inside the fifth power at the denominator

$$u = 3x^2-6x+4$$

$$\frac{du}{dx} = 6x-6 = 2(3x-3)$$

So $\frac{1}{2}\,du = (3x-3)\,dx$

Hence the integral becomes

$$= \int \frac{\frac{1}{2}\,du}{u^5} = \frac{1}{2}\int \frac{1}{u^5}\,du = \frac{1}{2}\int u^{-5}\,du$$

$$= \frac{1}{2}\left(\frac{1}{-4}u^{-4}\right) + C = -\frac{1}{8}\cdot\frac{1}{u^4} + C$$

$$= \boxed{-\frac{1}{8}\cdot\frac{1}{(3x^2-6x+4)^4} + C}$$

$\boxed{\#2}$ $\int x^3 \cdot \sqrt{x^2+5}\, dx$

set $u = x^2+5$; then $\frac{du}{dx} = 2x$

so $\frac{1}{2x} du = dx$. Finally observe that

$x^2 = u-5$. Hence

$$\int x^3 \cdot \sqrt{x^2+5}\, dx = \int x^2 \cdot x \cdot \sqrt{x^2+5} \cdot dx$$

$$= \int (u-5) \cdot \cancel{x} \sqrt{u} \cdot \frac{1}{2\cancel{x}} du = \int \frac{1}{2}(u-5) \cdot u^{\frac{1}{2}} du$$

$$= \int \left(\frac{1}{2} u^{3/2} - \frac{5}{2} u^{1/2}\right) du = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5} u^{5/2} - \frac{5}{\cancel{2}} \cdot \frac{\cancel{2}}{3} u^{3/2} + C$$

$$= \frac{1}{5} u^{5/2} - \frac{5}{3} u^{3/2} + C = \boxed{\frac{1}{5}(x^2+5)^{5/2} - \frac{5}{3}(x^2+5)^{3/2} + C}$$

$\boxed{\#3}$ $\int \frac{2ax+b}{ax^2+bx+c} \cdot dx$

set $u = ax^2+bx+c$

$\frac{du}{dx} = 2ax+b$; so $du = (2ax+b)\, dx$

Thus

$$\int \frac{2ax+b}{ax^2+bx+c}\,dx = \int \frac{du}{u} = \ln|u| + c$$

$$= \boxed{\ln|ax^2+bx+c| + c}$$

$\boxed{\#4}$ $$\int g'(x)\cdot e^{-g(x)}\cdot dx$$

Set $u = g(x)$; so $\frac{du}{dx} = g'(x)$

or $du = g'(x)\cdot dx$. Thus

$$\int g'(x)\cdot e^{-g(x)}\cdot dx = \int e^{-u}\cdot du =$$

$$= -e^{-u} + C = \boxed{-e^{-g(x)} + C}$$

$\boxed{\#5}$ $$\int_0^2 \frac{x}{x+2}\cdot dx$$ Set $u = x+2$;

$\frac{du}{dx} = 1$

so that $du = dx$. Also $x = u - 2$

Thus

$$\int_{0}^{2} \frac{x}{x+2}\, dx = \int_{2}^{4} \frac{u-2}{u} \cdot du$$

$x=2$ — $u = x+2$

$x=0$ $\longrightarrow$ $u = x+2$

$$\int_{2}^{4} \left(1 - \frac{2}{u}\right) du = \quad u - 2 \cdot \ln|u| \Big|_{2}^{4}$$

$$= (4 - 2\ln 4) - (2 - 2\ln(2))$$

$$= 4 - 2 \cdot \ln(2^2) - 2 + 2\ln(2)$$

$$= 4 - 4\ln(2) - 2 + 2\ln(2) = \boxed{2 - 2\ln(2)}$$

$$\approx \boxed{0.6137}$$

$\boxed{\#6}$ $\displaystyle\int_{\pi/3}^{\pi/2} \frac{\cos(z)}{\sin^2(z)}\, dz$

$u = \sin(z)$ as $\dfrac{du}{dz} = \cos(z)$ so

$du = \cos(z) \cdot dz$ $\qquad$ $\underline{f(z) = \cos(z)}$

After making the substitution

$u = \sin(z)$ and observing that

$$z = \frac{\pi}{3} \longrightarrow u = \sin(\pi/3) = \sqrt{3}/2$$

$$z = \pi/2 \longrightarrow u = \sin(\pi/2) = 1$$

we obtain

$$= \int_{\sqrt{3}/2}^{1} \frac{du}{u^2} = \int_{\sqrt{3}/2}^{1} u^{-2}\, du =$$

$$= -u^{-1}\Big|_{\sqrt{3}/2}^{1} = -\frac{1}{u}\Big|_{\sqrt{3}/2}^{1} =$$

$$= -1 - \left(-\frac{1}{\sqrt{3}/2}\right) = \boxed{\frac{2}{\sqrt{3}} - 1} \approx \boxed{0.1547}$$