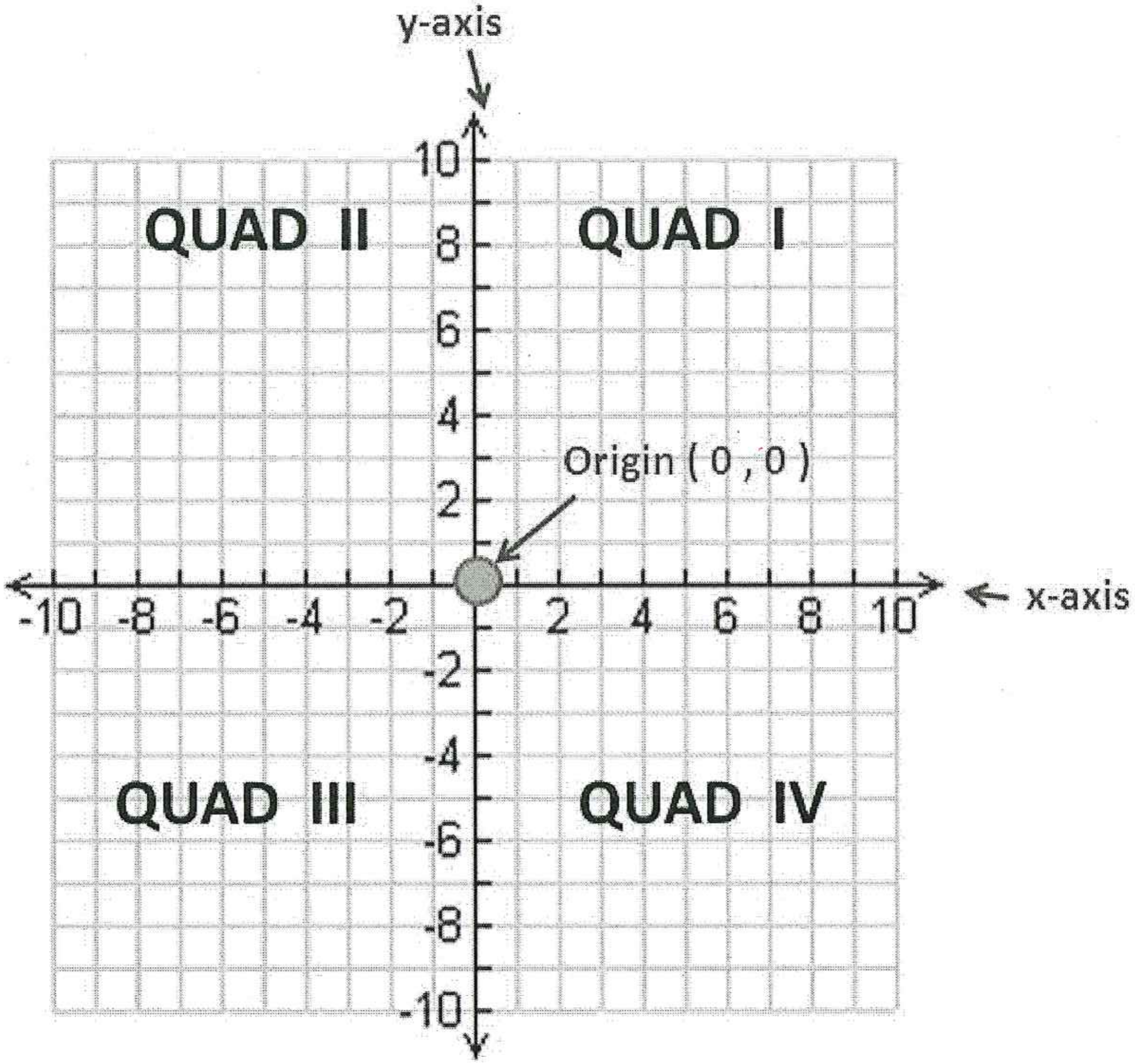
MA 162 : Finite Mathematics - Chapter 1

Linear Algebra

University of Kentucky

1.2 The Cartesian Coordinate Plane

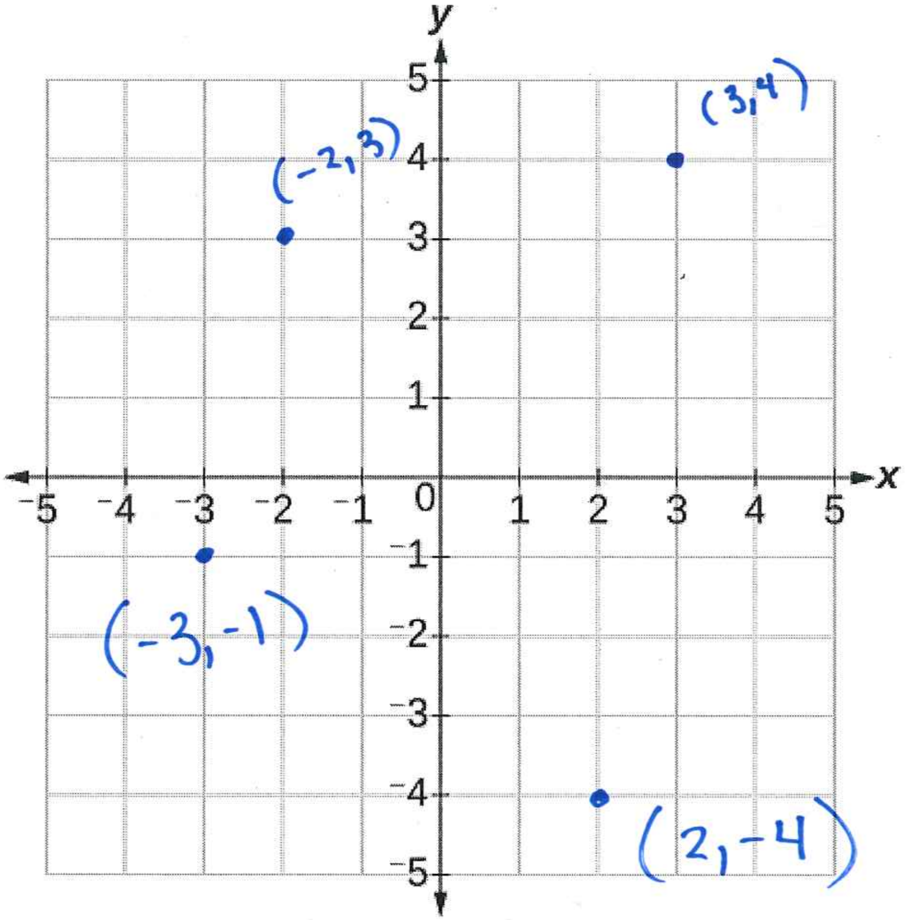
Cartesian coordinates are a pair of numbers (x, y) that tell the location of a point in the $xy-$plane.



1.2 Plotting Points In The Plane

Given a point (x, y), we can plot it in the plane.

Example 1: Plot the points $(3, 4), (-2, 3), (-3, -1)$, and $(2, -4)$.



1.2 Equations of Lines

- Given two points in the plane, we can connect them with a straight line. Conversely, if we are given the equation of a line, we can graph the line by finding two points that satisfy the equation.
- Equations whose graphs are lines are called **Linear Equations**.

Example 2: Graph the line $y = 2x + 1$.

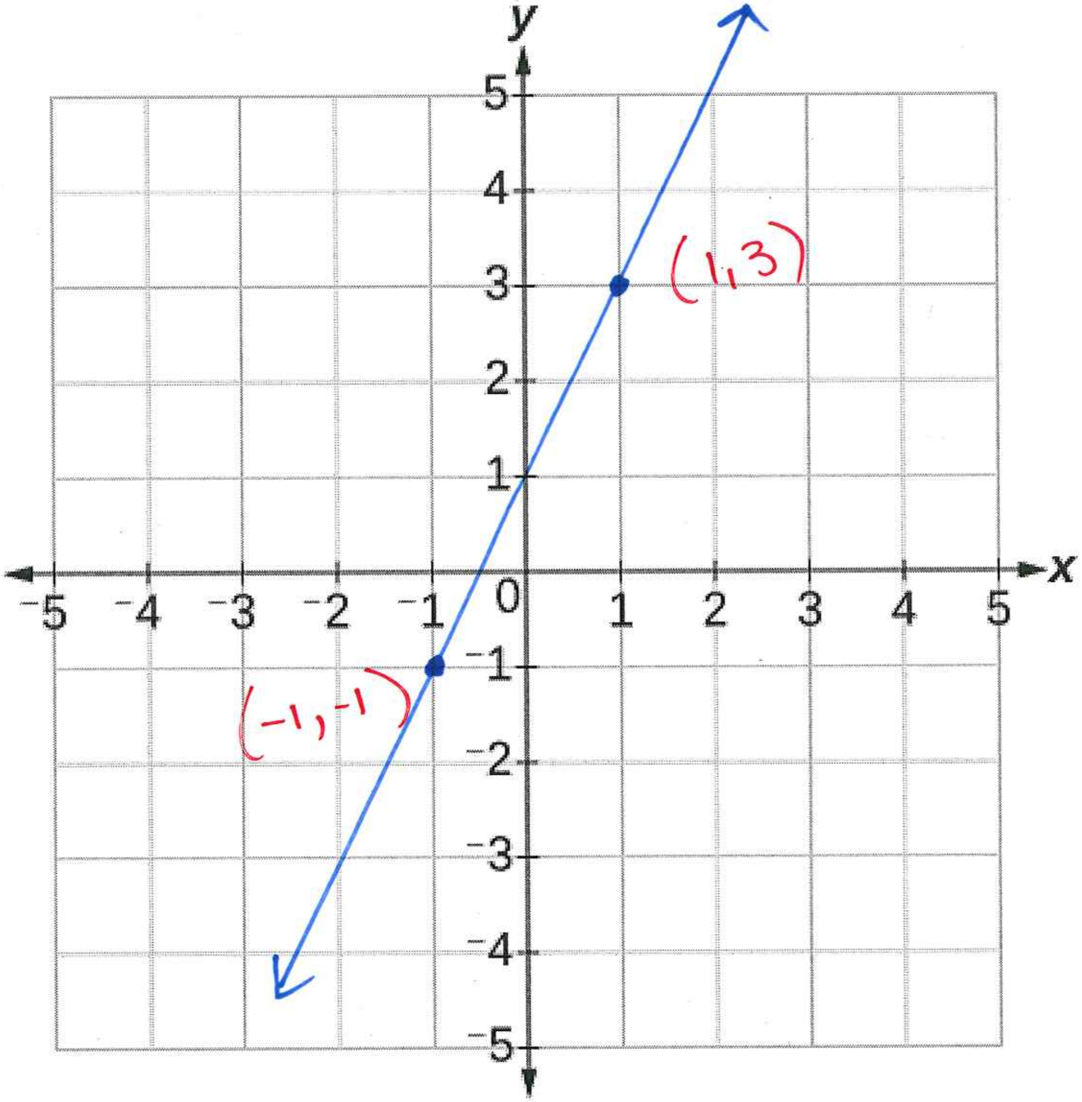
First we need to find two points (x, y) that satisfy the equation. Any two points will do, so we can pick any two values for x and find the corresponding y values. Lets use $x = -1$ and $x = 1$.

$x=1$: $y = 2(1)+1 = 3$ $(1,3)$

$x=-1$: $y = 2(-1)+1 = -1$ $(-1,-1)$

1.2 Example 2 Graphing a Line - Continued

We found the two points $(-1,-1)$ and $(1,3)$ satisfy the equation. Now we plot the points and connect them with a straight line.



1.2 x and y Intercepts

- The point $(x, 0)$ where the line crosses the x-axis is called the x-**intercept**.
- The point $(0, y)$ where the line crosses the y-axis is called the y-**intercept**.

If we are given a line in **General Form** $Ax + By = C$ for some numbers A, B, C where $A, B \neq 0$, then the easiest way to find two points that satisfy the equation is to find the intercepts.

- To find the x-intercept, replace $y = 0$ and solve for x.
- To find the y-intercept, replace $x = 0$ and solve for y.

1.2 Finding x and y Intercepts

Example 3: Graph the line $3x - 2y = 6$ by finding the intercepts.

x-int:

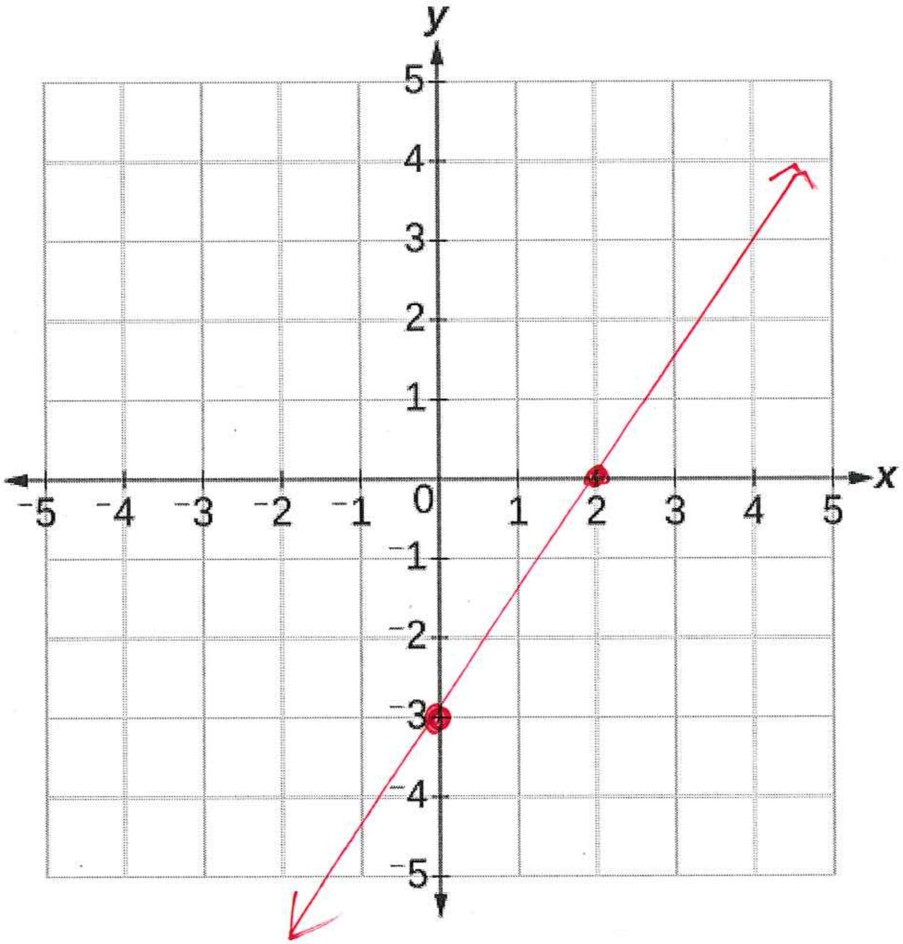
Set $y=0$
Solve for x.

$3x - 2(0) = 6$

$3x = 6$

$x = 2$

$(2,0)$



y-int:

Set $x=0$
Solve for y

$3(0) - 2y = 6$

$-2y = 6$

$y = -3$

$(0,-3)$

1.2 Horizontal and Vertical Lines

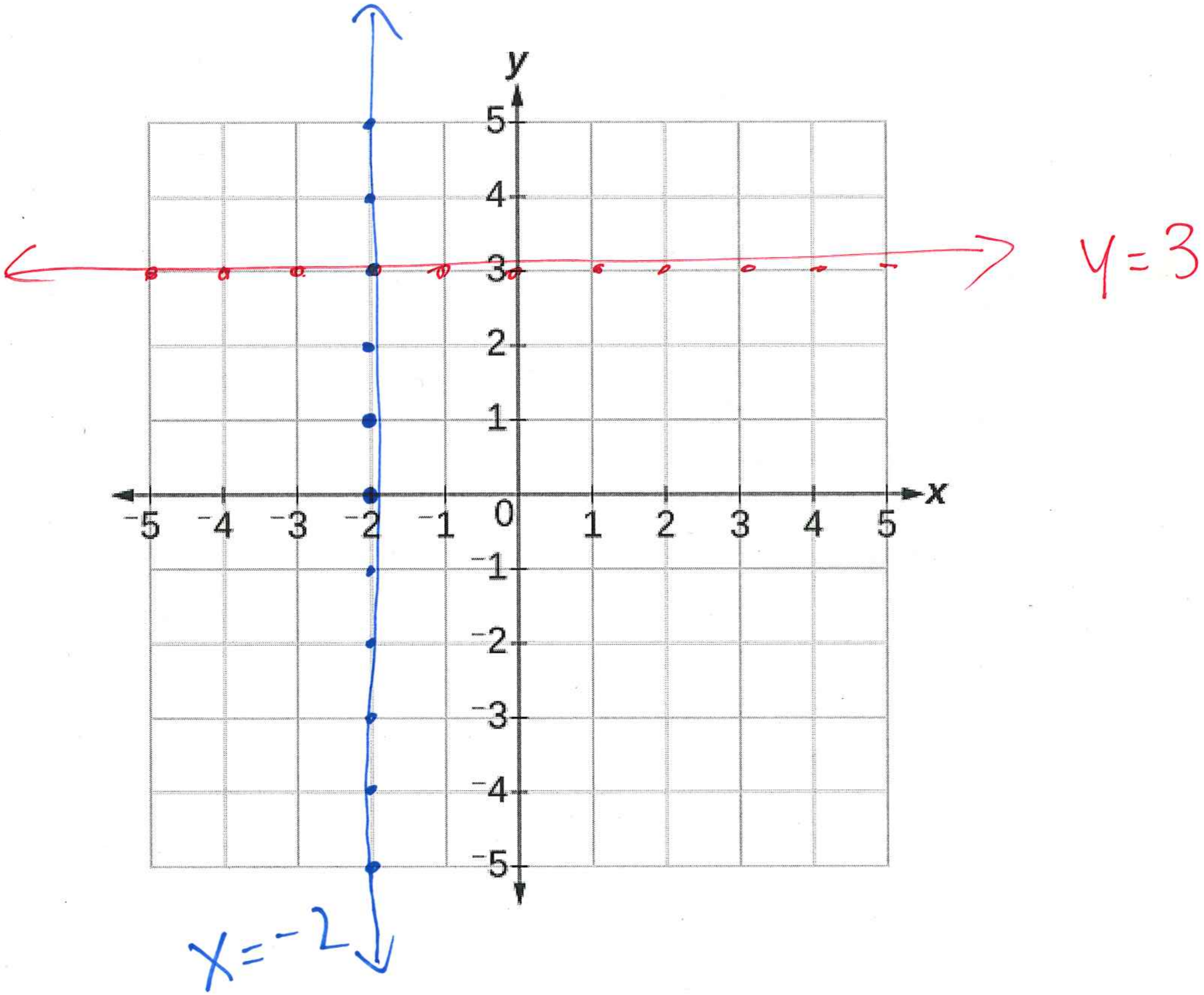
When the equation of a line involves only one variable, it is a horizontal or vertical line.

- The graph of the line $x = a$, where a is a constant, is a vertical line that passes through the point $(a, 0)$. Every point on this line has the x-coordinate a, regardless of the y-coordinate.

- The graph of the line $y = b$, where b is a constant, is a horizontal line that passes through the point $(0, b)$. Every point on this line has the y-coordinate b, regardless of the x-coordinate.

1.2 Horizontal and Vertical Lines

Example 4: Graph the lines $x = -2$ and $y = 3$.



1.3 Slope of a Line

- In the last section, we graphed lines by finding two points that satisfied the linear equation.
- Lines can also be graphed if we are given one point on the line and the "steepness" or "inclination" of the line.
- The number that refers to the "steepness"' of a line is the **slope** of a line.
- You may remember from algebra class that the slope of a line is given by the "rise" divided by the "run" or the vertical change divided by the horizontal change.

1.3 Slope of a Line

Definition: Let (x_0, y_0) and (x_1, y_1) be two different points on a line. Then the slope of the line is given by

$$\text{Slope} = m = \frac{y_1 - y_0}{x_1 - x_0}$$

Example 1: Find the slope of the line passing through $(2, 4)$ and $(3, -5)$.

(x_0, y_0)

(x_1, y_1)

$$m = \frac{-5-4}{3-2} = \frac{-9}{1} = \boxed{-9}$$

1.3 Zero Slope versus Undefined Slope

Example 2: Find the slope of the line passing through $(3, 4)$ and $(-2, 4)$

(x_0, y_0)

(x_1, y_1)

$$m = \frac{4-4}{-2-3} = \frac{0}{-5} = 0$$

Example 3: Find the slope of the line passing through $(3, 1)$ and $(3, -3)$.

(x_0, y_0)

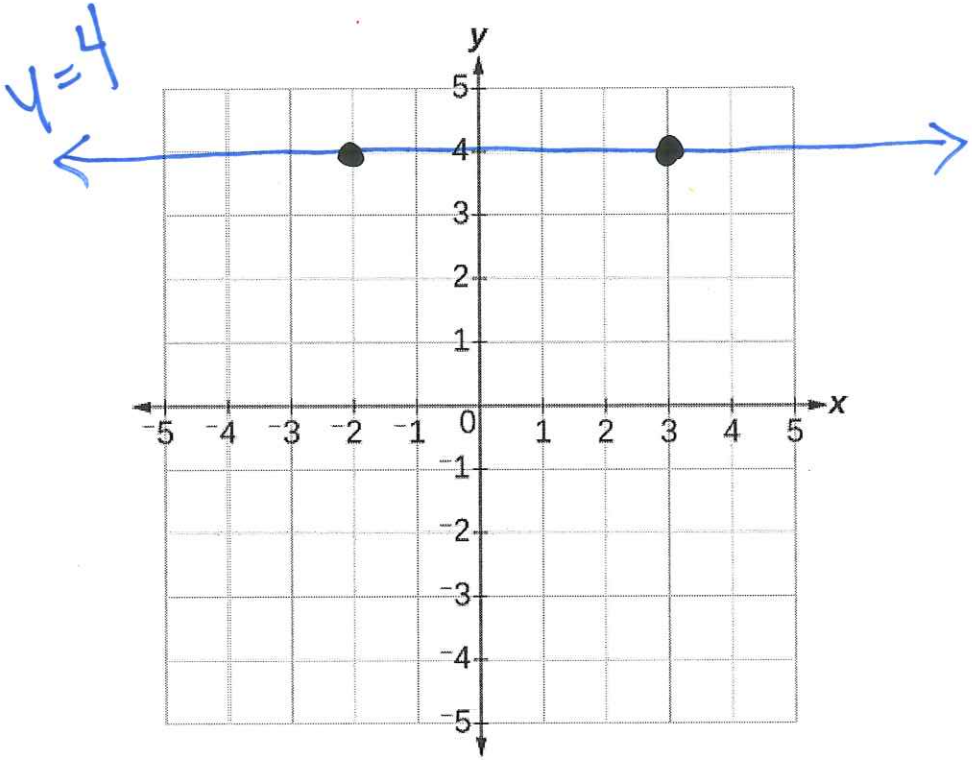
(x_1, y_1)

$$m = \frac{1-(-3)}{3-3} = \frac{4}{0}$$

Undefined Slope

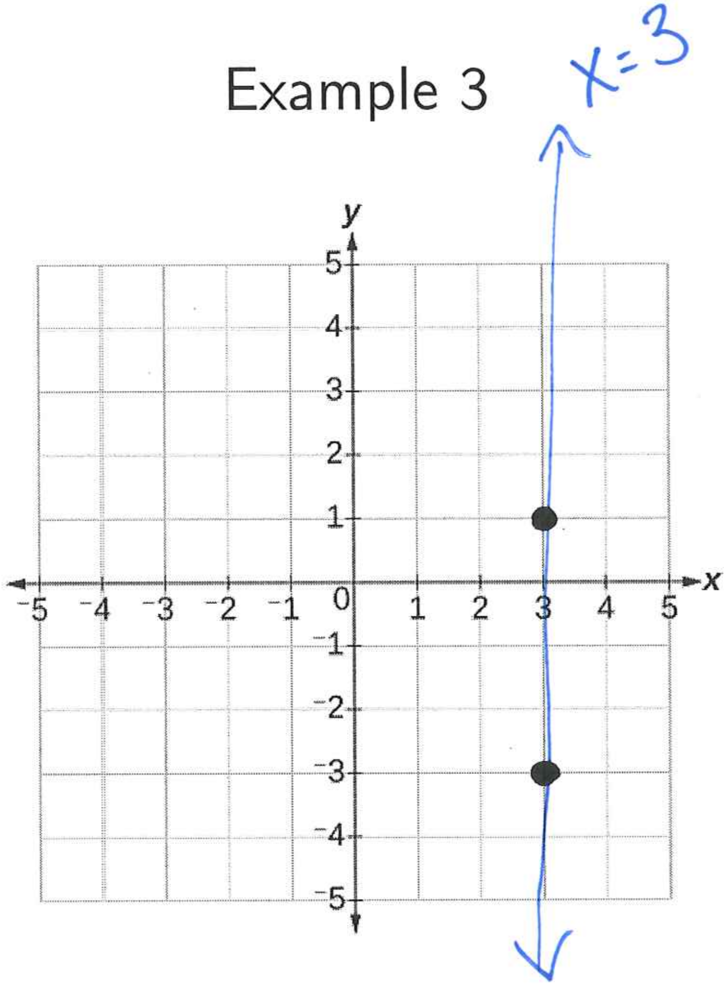
1.3 Zero Slope versus Undefined Slope

Example 2



Zero Slope

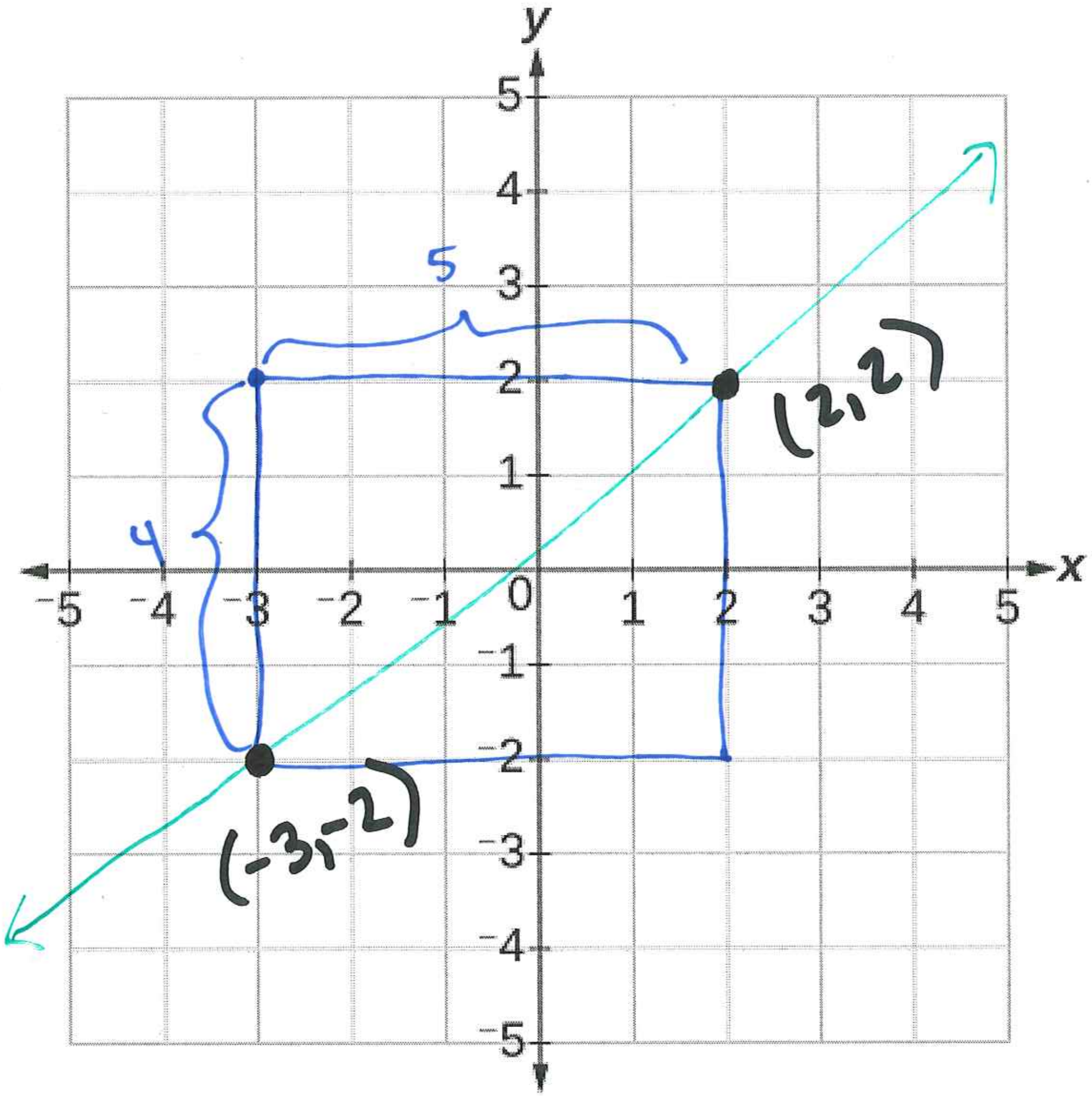
Example 3



Undefined Slope

1.3 Graphing Using Slope

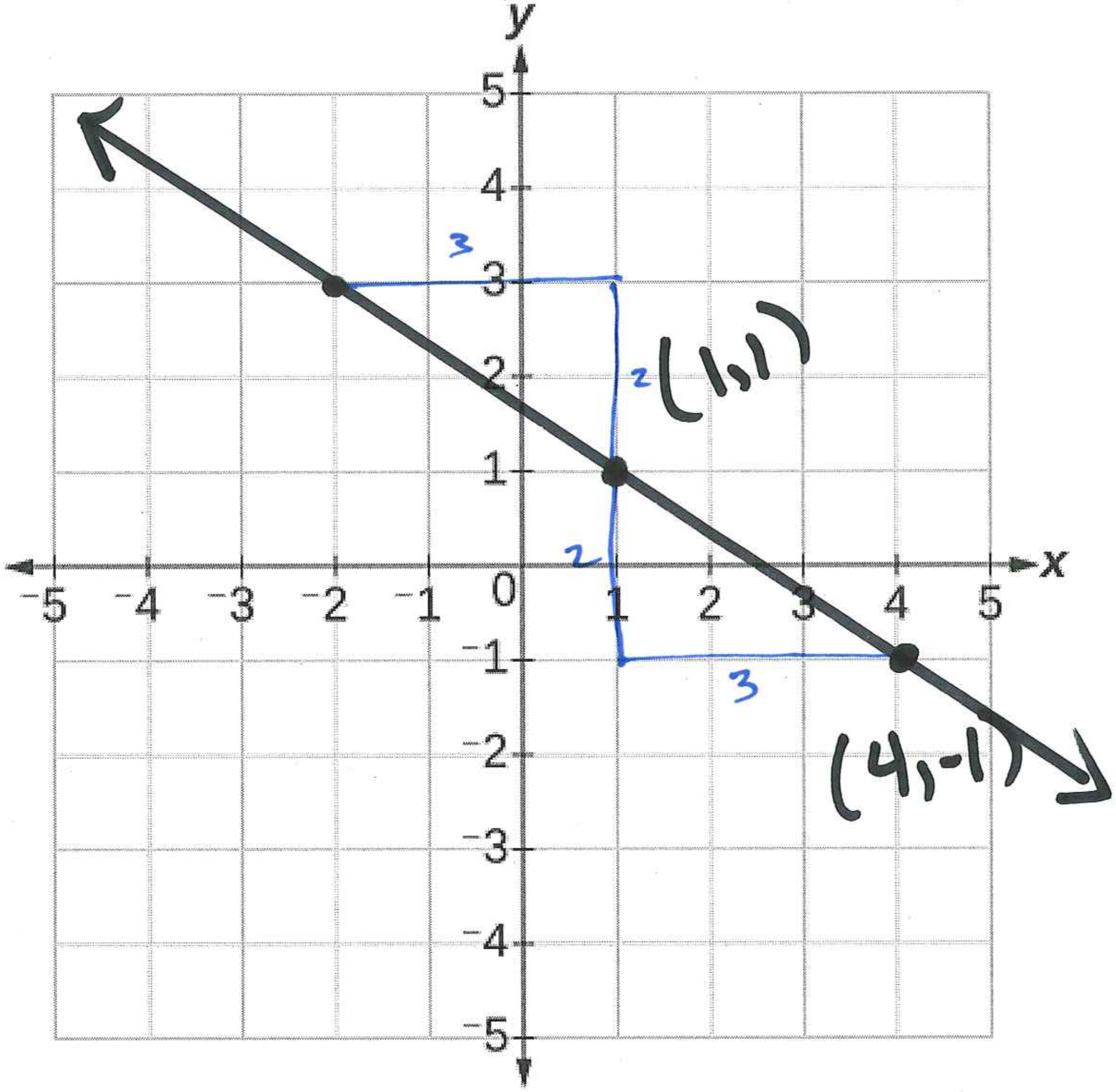
Example 4: Graph the line that passes through the point $(-3,-2)$ and has slope $m=\frac{4}{5}$.



1.3 Graphing Using Slope

Example 5: Graph the line that passes through the point $(1, 1)$ and has slope $m = -\frac{2}{3}$.

$-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$



1.3 Finding Slope Given an Equation

Example 6: Find the slope of the line with equation $3x + 4y = 6$.

Find any two points on the line

Let $x=0 \Rightarrow 3(0)+4y=6 \Rightarrow 4y=6 \Rightarrow \frac{3}{2}=y$

$(0,\frac{3}{2})$ y-int

Let $y=0 \Rightarrow 3x+4(0)=6 \Rightarrow 3x=6 \Rightarrow x=2$

$(2,0)$ x-int

$$m=\frac{\frac{3}{2}-0}{0-2}=\frac{\frac{3}{2}}{\frac{-2}{1}}=\frac{3}{2}\cdot\frac{1}{-2}=\boxed{-\frac{3}{4}}$$

1.3 Finding Slope Given an Equation

Example 7: Find the slope of the line with equation $y = 4x + 1$.

$x=0 \Rightarrow y = 4(0)+1 = 1 \quad (0,1)$

$x=1 \Rightarrow y = 4(1)+1 = 5 \quad (1,5)$

$$m = \frac{5-1}{1-0} = \frac{4}{1} = \boxed{4}$$

y-int is $(0,\boxed{1})$

1.3 Finding Slope Given an Equation

- In the last example, we found the slope of $y = 4x + 1$ to be 4 and the y-intercept to be $(0, 1)$.

- It is not a coincidence that when the equation of the line is solve for y, then the coefficient of x is the slope and the constant term is the y-intercept.

Equation	Slope	y-intercept
$y = 2x - 3$	2	$(0, -3)$
$y = -3x + 7$	-3	$(0, 7)$
$y = \frac{1}{3}x - 2$	$\frac{1}{3}$	$(0, -2)$

1.3 Finding Slope Given an Equation

Example 8: Find the slope of the line with equation $3x + 4y = 6$

Solve $\boxed{3x}+4y=6$ for y

$$4y = -3x + 6$$

$$y = -\frac{3}{4}x + \frac{3}{2}$$

$m = -\frac{3}{4}$ yint is $\left(0, \frac{3}{2}\right)$

1.3 Finding Slope Given an Equation

Example 9: Find the slope of the line with equation

$$8x + 7y + 23 = 2x - 3y + 7$$

$$10y = -6x - 16$$

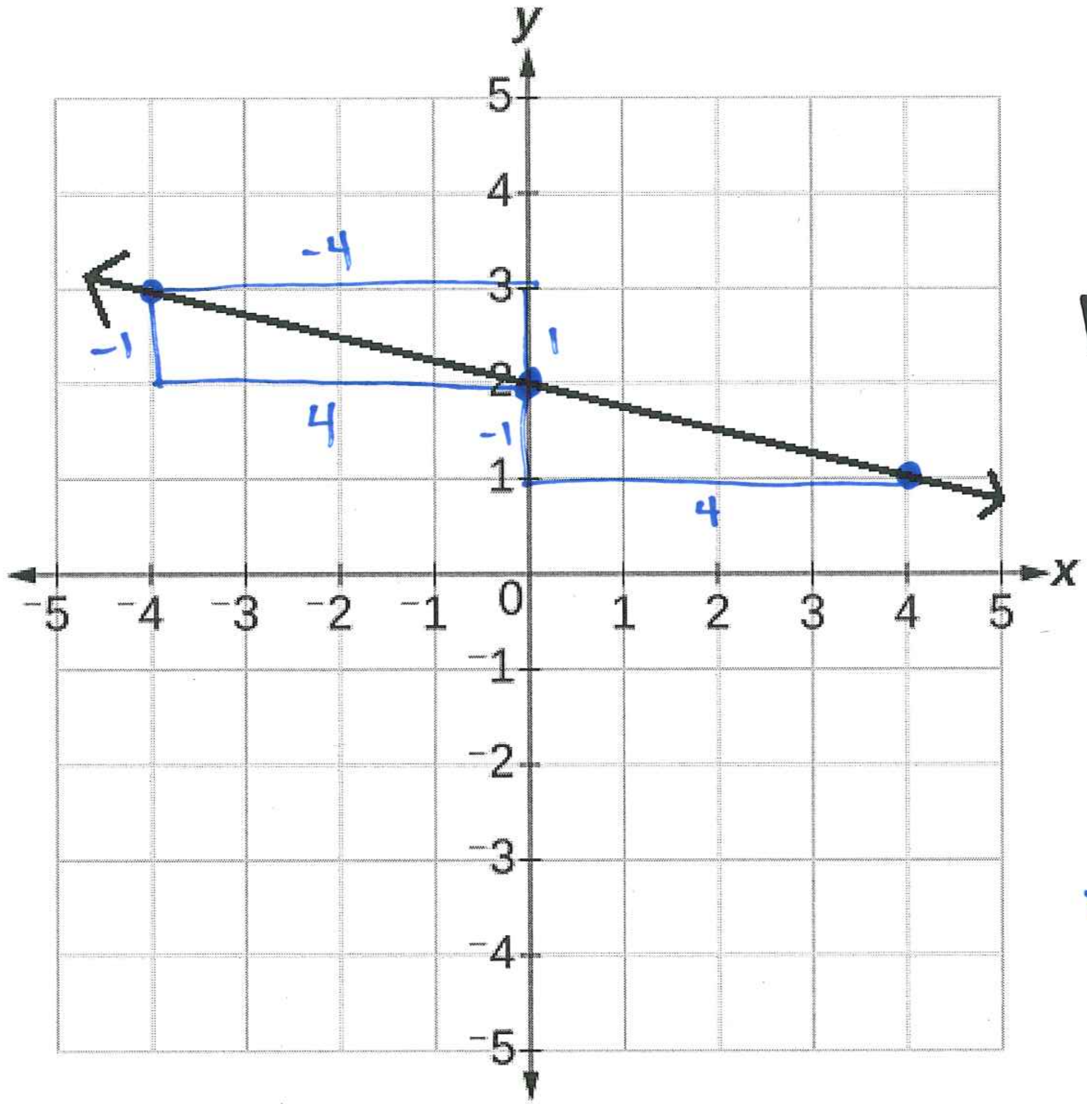
$$y = -\frac{6}{10}x - \frac{16}{10}$$

$$y = -\frac{3}{5}x - \frac{8}{5}$$

Slope $= m = -\frac{3}{5}$, y-int is $\left(0, \frac{-8}{5}\right)$

1.3 Slope and y-Intercept From a Graph

Example 10: Find the slope and y-intercept of the given line.



y-int is (0,2)

$$m = \frac{-1}{4} = \frac{1}{-4} = -\frac{1}{4}$$

$$y = mx + \underline{b}$$

$$y = -\frac{1}{4}x + 2$$

1.4 Determining the Equation of a Line

There are three common forms for the equation of a line.

- **General Form**: $Ax + By = C$ where A, B, C are real numbers.
- **Slope Intercept Form**: $y = mx + b$ where m is the slope and b is the y-coordinate of the y-intercept.
- **Point Slope Form**: $y - y_1 = m(x - x_1)$ where m is the slope and the line passes through the point (x_1, y_1).

Each form is useful depending on the information we are given or the information we are asked to find.

1.4 Slope Intercept Form

Example 1: Find the equation of the line with slope 5 and y-intercept $(0, -2)$.

$y = mx + b$

b

$y = 5x - 2$

1.4 Point Slope Form

Example 2: Find the equation of the line with slope 5 that passes through the point $(3, -2)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = 5(x - 3)$$

$$y + 2 = 5x - 15$$

$$y = 5x - 17$$

1.4 Determining the Equation of a Line

$y = mx + b$

Example 3: Find the slope intercept form of the line that passes through the points $(2, 4)$ and $(5, 10)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-4}{5-2} = \frac{6}{3} = 2$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - 2)$$

$$y - 4 = 2x - 4$$

$$\boxed{y = 2x}$$

$$y - y_1 = m(x - x_1)$$

$$y - 10 = 2(x - 5)$$

$$y - 10 = 2x - 10$$

$$\boxed{y = 2x}$$

Example 4: Find the general form of the line that passes through the points (2, 4) and (5, 10).

$$Ax + By = C$$

Ex 3 ⇒ $y = 2x$

$$\boxed{-2x + y = 0}$$

1.4 Determining the Equation of a Line

Example 5: Write the equation of the line $5x - 3y = 7$ in slope intercept form and determine the slope and *y*-intercept.

Solve for y

$$5x - 3y = 7$$

$$\underset{-5x}{} \qquad \underset{-5x}{}$$

$$-3y = -5x + 7$$

$$y = \frac{5}{3}x - \frac{7}{3}$$

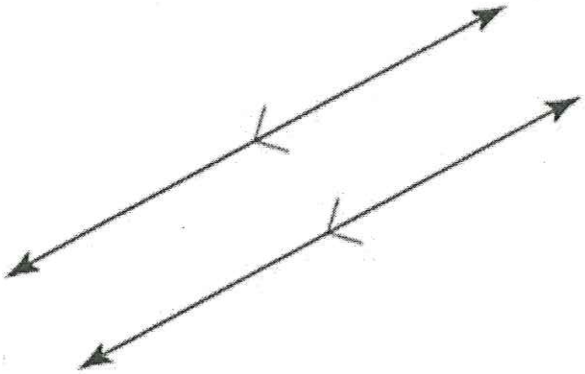
$m = \text{slope} = \frac{5}{3}$ y-int $= (0, -\frac{7}{3})$

1.4 Parallel and Perpendicular Lines

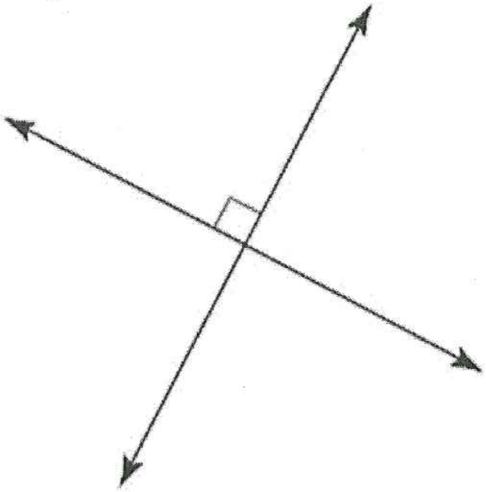
- Parallel lines have the same slope.

- If the slope of a line is $\frac{a}{b}$, then the slope of any perpendicular line has slope $-\frac{b}{a}$. (We need $a, b \neq 0$).

Parallel lines



Perpendicular lines



1.4 Parallel Lines

Example 6: Find the equation of the line that passes through the point $(-3, 5)$ and is parallel to the line $y = 4x + 1$.

(x_1, y_1)

same slope

$\uparrow$ $m = 4$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 4(x - (-3))$$

$$y - 5 = 4(x + 3)$$

$$y - 5 = 4x + 12$$

$$y = 4x + 17$$

1.4 Perpendicular Lines

Example 7: Find the equation of the line that passes through the point $(-5, 5)$ and is perpendicular to the line $y = 3x + 3$.

$m = 3 = \frac{3}{1}$

Slope of the line we want is $-\frac{1}{3}$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{1}{3}(x - (-5))$$

$$y - 5 = -\frac{1}{3}(x + 5)$$

$$y - 5 = -\frac{1}{3}x - \frac{5}{3}$$

$$y = -\frac{1}{3}x - \frac{5}{3} + \frac{15}{3}$$

$$y = -\frac{1}{3}x + \frac{10}{3}$$

1.4 Determining the Equation of a Line

Example 8: Find the value of k so that the line containing the points $(4, k)$ and $(8, 7)$ is perpendicular to the line $y = \underset{m=}{\frac{1}{7}}x + 4$.

the line through $(4,k)$ & $(8,7)$ should have slope $= -7$.

Need $m = \boxed{\frac{7-k}{8-4} \overset{?}{=} -7}$

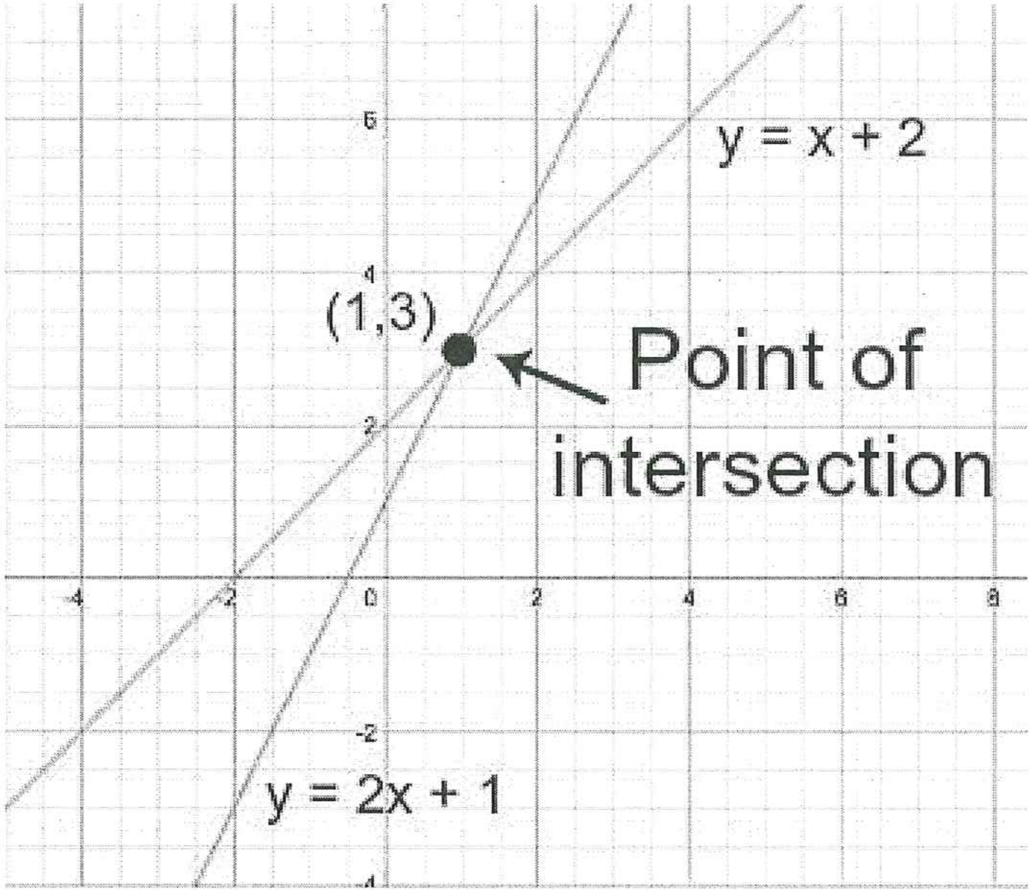
$$\frac{7-k}{4} = -7$$

$$\Rightarrow \underset{+k+28}{7-k} = \underset{+k+28}{-28} \Rightarrow \boxed{35 = k}$$

Systems of Equations

- We have already seen that any point on a line satisfies the equation of the line.
- Suppose we are given two lines and are asked to find the point where they intersect. In this case, we are looking for a point that satisfies both equations.
- The process of finding the point of intersection of two lines is called *solving a system* of two equations with two variables.

Systems of Equations



To have a single point of intersection, the lines can not be parallel or both be the same line in different forms. As long as the lines have different slopes, there will always be a single point of intersection.

Systems of Equations

- We have already seen that any point on a line satisfies the equation of the line.
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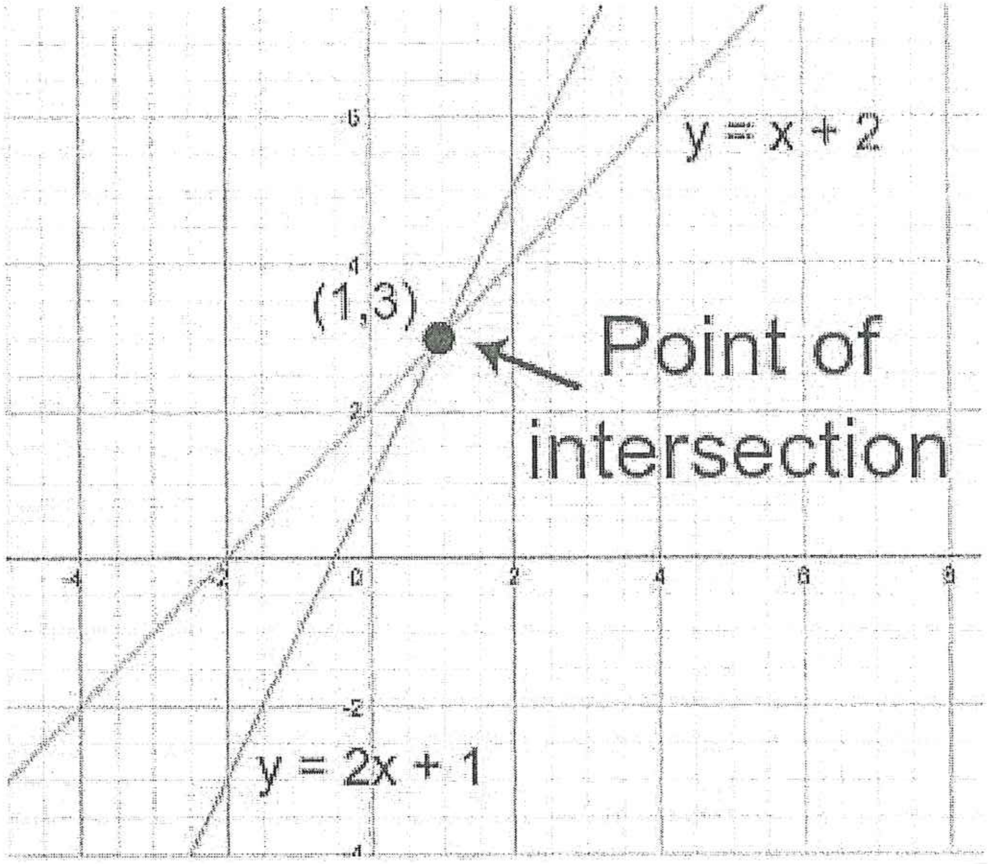
Systems of Equations



To have a single point of intersection, the lines can not be parallel or both be the same line in different forms. As long as the lines have different slopes, there will always be a single point of intersection.

Systems of Equations

There are two methods we will use to solve a system of equations.

- **Substitution**
  - Solve one of the equations for y.
  - Substitute the result into the other equation.
  - Solve for x.
  - Plug the value for x into either equation to find y.

Note: We could also start by solving one of the equations for x and going through a similar process.

Systems of Equations: Substitution Method

Example 1: Find the intersection of the lines $\underline{y = 2x}$ and $y = -x + 6$ using the substitution method.

$$y = -x + 6$$

$$2x = -x + 6$$

$$3x = 6$$

$$x = 2$$

$$y = 2x$$

$$y = 2(2) = 4$$

$$\boxed{(2,4)}$$

Systems of Equations

- **Elimination**
  - Write both equations in general form.
  - Multiply the equations by constants so that the coefficients of y are opposites of each other.
  - Add the equations to eliminate the y variable.
  - Solve for x.
  - Plug the value for x into either equation to find y

Note: We could also eliminate the x variable and solve for y.

Systems of Equations: Elimination Method

Example 2: Find the intersection of the lines $-8x + 7y = 64$ and $-7x + y = 15$ using the elimination method.

$$-8x+7y=64$$

$$(-7x+y=15)(-7)$$

$$\begin{array}{rl} & -8x+\cancel{7y}=64 \\ + & 49x-\cancel{7y}=-105 \\ \hline & 41x=-41 \end{array}$$

$$\underline{\underline{x=-1}}$$

$$-7x+y=15$$

$$-7(-1)+y=15$$

$$7+y=15$$

$$\underline{\underline{y=8}}$$

$$\boxed{(-1,8)}$$

Systems of Equations

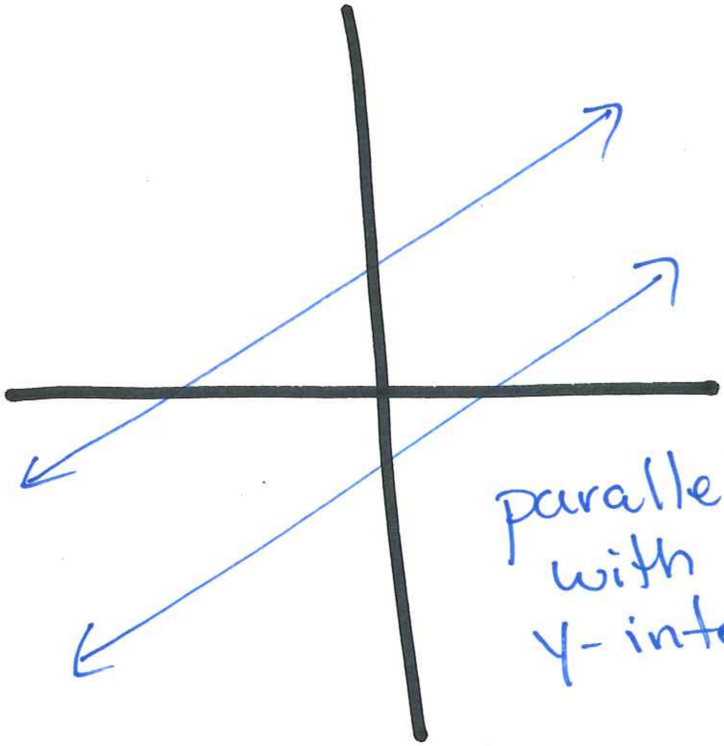
Example 3: Solve the system of equations using either method.

$$\begin{cases} \left(2x-6y=5\right)\cdot(3) \\ -3x+9y=-9\cdot(2) \end{cases}$$

$$\begin{array}{r} 6x-18y=15 \\ \underline{-6x+18y=-18} \\ 0=-3 \end{array}$$

?

No Solution!



parallel lines with different y-intercepts

Systems of Equations: No Solution

If two lines are parallel with different y-intercepts, then there will be no solution.

Example 4: Show the following system of equations has no solution.

$$\begin{cases} (2x - 3y = 5)\cdot(2) \\ -4x + 6y = -9 \end{cases}$$

$$\begin{array}{r} 4x - 6y = 10 \\ -4x + 6y = -9 \\ \hline 0 = 1 \end{array}$$

No Solution!

Systems of Equations: Infinite Solutions

If two lines are parallel with the same y-intercept, then there will be infinitely many solutions.

Example 5: Show the following system of equations has infinitely many solutions.

$$\begin{cases} (2x - 3y = 5)\cdot(2) \\ -4x + 6y = -10 \end{cases}$$

Solutions are points (x,y)

$$4x - 6y = 10$$
$$-4x + 6y = -10$$
$$\overline{\qquad 0 = 0 \qquad}$$

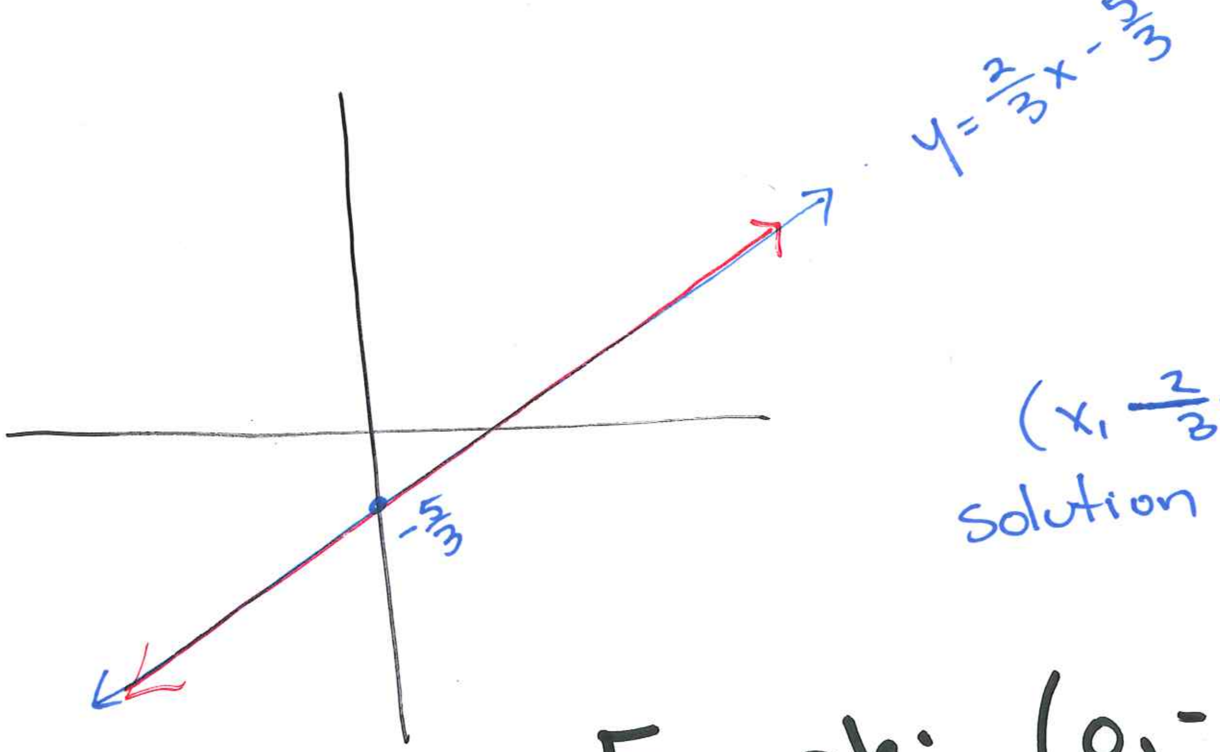
Infinitely many Solutions

Solve $-4x + 6y = -10$ for y.

$$y = \frac{2}{3}x - \frac{5}{3}$$

$$\boxed{(x,y) = \left(x, \frac{2}{3}x - \frac{5}{3}\right)}$$

parameterizing the solutions



$\left(x, \frac{2}{3}x - \frac{5}{3}\right)$ is the solution for every x-value.

Example: $\left(0, -\frac{5}{3}\right)$ $x = 0$

$(1, -1)$ $x = 1$

$\left(2, -\frac{1}{3}\right)$ $x = 2$

Systems of Equations: No Solution

Example 6: Determine all values of h and k for which the system has no solution.

$$\begin{cases} 2x + ky = 1 \Longrightarrow y = -\frac{2}{k}x + \frac{1}{k} \\ -3x + 8y = h \Longrightarrow y = \frac{3}{8}x + \frac{h}{8} \end{cases}$$

Same slope
different y-int's

Same slope:

Need $-\frac{2}{k} = \frac{3}{8}$

$-16 = 3k$

$\boxed{-\frac{16}{3} = k}$

Different y-int's

Need $\frac{1}{k} \neq \frac{h}{8}$

$$\frac{1}{-\frac{16}{3}} \neq \frac{h}{8}$$

$$-\frac{3}{16} \neq \frac{h}{8} \Rightarrow -\frac{24}{16} \neq h \Rightarrow \boxed{-\frac{3}{2} \neq h}$$

Systems of Equations

- A system of equations with no solution is called **inconsistent**.
- A system of equations with at least one solution is called **consistent**.

one or infinite

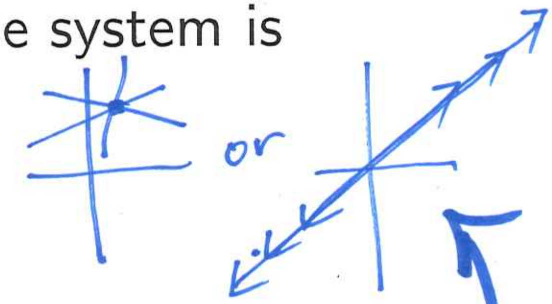
- A consistent system with infinitely many solutions is called **dependent**.

Systems of Equations

- A system of equations with no solution is called **inconsistent**.
- A system of equations with at least one solution is called **consistent**. Exactly one or Infinitely many
- A consistent system with infinitely many solutions is called **dependent**.

Systems of Equations: Consistent

Example 7: Determine all values of k for which the system is consistent.



$$\begin{cases} 9x - 15y = 6 \\ 18x - 30y = k \\ 12x - 20y = 8 \end{cases}$$

slope $= \frac{3}{5}$

y-int $= (0, -\frac{2}{5})$

→ Same line

slope $= \frac{3}{5}$

Need $18x - 30y = k$ to have y-int $(0, \frac{2}{5})$

y-int $= (0, \frac{k}{-30}) \Rightarrow -\frac{k}{30} = -\frac{2}{5}$

$K = \frac{60}{5} = \boxed{12}$

Systems of Equations: Fractions :(

Example 8: Solve the system of equations.

$$\begin{cases} \left(\frac{1}{4}x - \frac{1}{3}y = \frac{1}{6}\right)(12) \\ \left(-\frac{1}{2}x - \frac{2}{3}y = -\frac{2}{1}\right)(6) \end{cases}$$

$$\begin{cases} \cancel{3x} - 4y = 2 \\ \cancel{-3x} - 4y = -12 \end{cases}$$

$$-8y = -10$$

$$y = \boxed{\frac{5}{4}}$$

$$3x - 4y = 2$$

$$3x - \cancel{4}\left(\frac{5}{\cancel{4}}\right) = 2$$

$$3x - 5 = 2$$

$$3x = 7$$

$$x = 7/3$$

$$\boxed{(x,y) = (7/3, 5/4)}$$

1.5 Applications

Example 1: It costs \$90 to rent a car driven 100 miles and \$140 for one driven 200 miles. If x is the number of miles driven and y the total cost of the rental, write the cost function.

$(x, y) =$ (# of miles, cost of rental)

We are given 2 points $\underline{(100, 90)}$ & $(200, 140)$

$$m = \frac{140-90}{200-100} = \frac{50}{100} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 90 = \frac{1}{2}(x - 100)$$

$$y - 90 = \frac{1}{2}x - 50 \rightarrow y = \frac{1}{2}x + \underline{\underline{40}}$$

1.5 Applications

Example 2: You are offered two different sales jobs. The first company offers a straight commission of 4% of the sales. The second company offers a salary of \$270 per week plus 3% of the sales. How much would you have to sell in a week in order for the straight commission offer to be at least as good?

x = amount sold in a week

Income from 1st job = Income from 2nd job.

$$.04x = 270 + .03x$$

$$-.03x \qquad\qquad -.03x$$

$$.01x = 270$$

$$x = 27000$$

1.5 Applications

Example 3: A firm producing widgets has to pay initial costs of \$10,725 for machinery and building rental. They have determined that it cost \$0.20 to produce a single widget. If they sell the widgets for \$1.50, how many widgets must they sell to break even? Let x be the number of widgets.

- Find cost function

Total cost = Fixed cost + (variable cost) x

$$C(x) = 10725 + .2x$$

- Find revenue function

Revenue = (# of widgets sold)(selling price per unit)

$$R(x) = 1.50x$$

Example 3 continued

- Find profit function

Profit = Revenue - Cost

$P(x) = R(x) - C(x)$

$P(x) = 1.5x - (10725 + .2x)$

- Find break even quantity

$= 1.3x - 10725$

Set

$P(x) = 0$ and solve for x

$1.3x - 10725 = 0$

$1.3x = 10725$

$x = 8250$ widgets

1.5 Applications

Example 4: Whackemhard Sports is planning to introduce a new line of tennis rackets. The fixed costs for the new line are $25,000 and the variable cost of producing each racket is $60. If the racket sells for $80, find the number of rackets that must be sold in order to break even.

x = # of rackets

$C(x) = 25000 + 60x$

$R(x) = 80x$

$P(x) = R(x) - C(x) = 20x - 25000$

$20x - 25000 = 0$

$20x = 25000$

$x =$

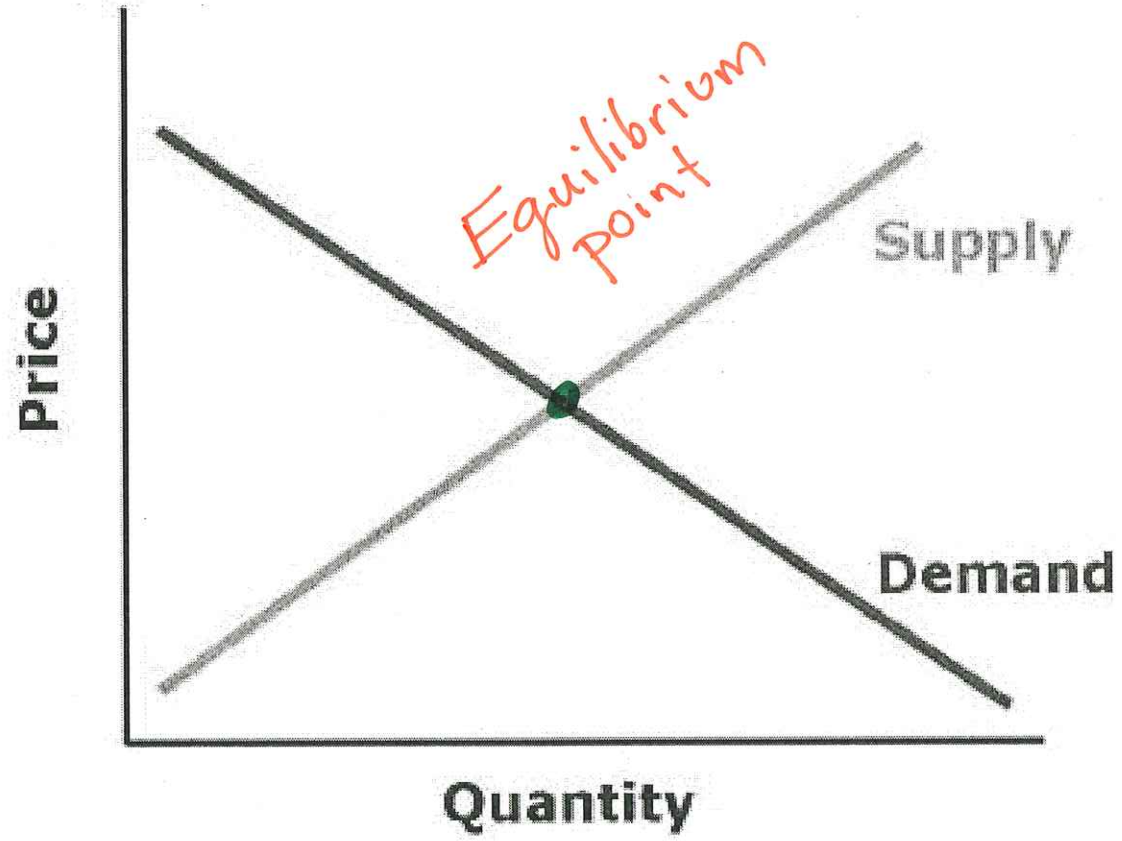
1.5 Applications (Supply and Demand)

Suppose a company is producing and supplying an item. The item is then purchased by a customer. Let x be the number of items produced or purchased and let p be the price of sale or purchase.

- The company will continue to produce more items as long as the price to produce an item is less than the price someone will pay for it. The quantity supplied will increase as long as the price of sale increases.
- A consumer will be less inclined to buy an item at a higher price and will buy more items at a lower price. The quantity demanded will increase as the price decreases.

1.5 Applications (Supply and Demand)

We are in the linear algebra section, so we assume that the relationship between price and quantity is linear for both the supply and demand equations.



1.5 Applications (Supply and Demand)

Example 5: The supply equation for a product is $p = 300x + 9000$, and the demand equation is $p = -100x + 14000$, where p represents the price and x the number of items. At what price will the supply equal the demand, and how many items will be produced at that price?

Find the equilibrium point (x, p)

Solve $300x + 9000 = -100x + 14000$

$$400x = 5000$$

$$\boxed{x = 12.5}$$

$$P = 300x + 9000$$

$$P = 300(12.5) + 9000 = \boxed{\underline{12750} = P}$$

1.5 Applications (Supply and Demand)

Example 6: The demand equation for the TI-83 graphing calculator is $x + 6p - 897 = 0$, where x is the quantity demanded per week and p is the unit price in dollars. The supply equation is $x - 17p + 805 = 0$, where x is the quantity the supplier will make available in the market each week when the wholesale price is p dollars each. Find the equilibrium quantity and the equilibrium price for the calculators.

Solve $\begin{cases} x+6p-897=0 \\ -(x-17p+805=0) \end{cases}$

$$\overline{\qquad 23p-1702=0 \qquad}$$

$$\Rightarrow 23p=1702$$

$$\boxed{p=74}$$

$$x+6(74)-897=0$$

$$x+444-897=0$$

$$x-453=0 \quad \Rightarrow \quad \boxed{x=453}$$

1.5 Applications

Example 7: A total of \$4,000 was invested, part at 5% and part at 7% simple interest. If the total yearly interest amount is \$200, how much was invested at 7%?

x = amount invested at 5%

y = amount invested at 7%

$x + y = 4000$ ← amount invested

$.05x + .07y = 200$ ← interest earned.

$$\Rightarrow \begin{array}{rl} -.05x - .05y &= -200 \\ + \quad .05x + .07y &= 200 \\ \hline .02y &= 0 \\ y &= 0 \end{array}$$

$x = 4000$

$(x, y) = (4000, 0)$

1.6 More Applications

Example 1: The Lakers scored a total of 80 points in a basketball game against the Bulls. The Lakers made a total of 37 baskets. How many 2 point shots did the Lakers make? How many 3 point shots did the Lakers make?

x = # of 2 point shots made

y = # of 3 point shots made

$$\begin{aligned} x+y &= 37 \\ 2x+3y &= 80 \end{aligned} \implies \begin{array}{r} -2x-2y = -74 \\ + \quad 2x+3y = 80 \\ \hline y = 6 \end{array}$$

$$x+6 = 37$$

$$\implies x = 31$$

$$(x,y) = (31,6)$$

1.6 More Applications

Example 2: Tickets for admission to a high school football game cost \$3 for students and \$5 for adults. During one game, \$2995 was collected from the sale of 729 tickets. Write and solve a system of linear equations to find the number of tickets sold to students and the number of tickets sold to adults.

x = # of tickets sold to students

y = # of tickets sold to adults

$$x+y=729 \xrightarrow{\times -3} -3x-3y=-2187$$

$$3x+5y=2995 \qquad 3x+5y=2995$$

$$2y=808$$

$$y=404$$

$$x+404=729$$

$$x=325$$

$$(x,y)=(325,404)$$

1.6 More Applications

Example 3: A test has twenty questions worth 100 points. The test consists of True/False questions worth 3 points each and multiple choice questions worth 11 points each. How many of each type of question are on the test?

x = # of T/F questions

y = # of multiple choice

$$\begin{aligned} x+y&=20 \\ 3x+11y&=100 \end{aligned} \qquad \xrightarrow{x\,-3} \qquad \begin{aligned} -3x-3y&=-60 \\ 3x+11y&=100 \\ \hline 8y&=40 \\ y&=5 \end{aligned}$$

$$x+5=20$$
$$x=15$$

$$(x,y)=(15,5)$$

1.6 More Applications

Example 4: Margie is responsible for buying a week's supply of food and medication for the dogs and cats at a local shelter. The food and medication for each dog costs twice as much as those supplies for a cat. She needs to feed 164 cats and 24 dogs. Her budget is $4240. How much can Margie spend on each dog for food and medication?

x = amount spent on each cat

y = amount spent on each dog

$$y = \boxed{2x}$$

$$164x + 24y = 4240$$

$$164x + 48x = 4240$$

$$212x = 4240$$

$$x = 20 \Rightarrow y = 40$$

$$\Rightarrow (x, y) = (20, 40)$$

1.6 More Applications

Example 5: Jake's Surf Shop rents surfboards for $6.00 plus $3.00 per hour. Ritas rents them for $9.00 plus $2.50 per hour. Which is cheaper for which time intervals? $x = \#$ of hours

Cost for Jakes

$$= 6.00 + 3.00x$$

Cost for Ritas

$$= 9.00 + 2.50x$$

Find when they are equal

$$6 + 3x = 9 + 2.5x$$

$$.5x = 3$$

$$x = 6$$

For less than 6 hours Jakes charges less

For more than 6 hours Ritas charges less.