MA 213 — Calculus III Fall 2016
Exam 1 Sep. 21, 2016

Name: ____________________________

Section: ____________________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:**
  Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:**
  Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		32
5		8
6		10
7		10
8		10
9		10
10		10
11		10
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. (8 points) Let

$$\mathbf{a} = \mathbf{i} - \mathbf{j}, \quad \mathbf{b} = \mathbf{j} - \mathbf{k}, \quad \mathbf{c} = \mathbf{k} - \mathbf{i}.$$

Compute

$$\mathbf{w} = \mathrm{proj}_{\mathbf{a}}\mathbf{b} + \mathrm{proj}_{\mathbf{b}}\mathbf{c} + \mathrm{proj}_{\mathbf{c}}\mathbf{a}.$$

A. $\mathbf{w} = \mathbf{0}$.

B. $\mathbf{w} = \mathbf{i} + \mathbf{j} + \mathbf{k}$.

C. $\mathbf{w} = -\mathbf{i} - \mathbf{j} - \mathbf{k}$.

D. $\mathbf{w} = \frac{1}{2}(\mathbf{i} + \mathbf{j} + \mathbf{k})$.

E. $\mathbf{w} = -\frac{1}{2}(\mathbf{i} + \mathbf{j} + \mathbf{k})$.

2. (8 points) Let $P = (1, 2, 3), Q = (2, 6, 6), R = (2, 2, 5), S = (4, 0, 5)$. Compute the area A of the the triangle $\triangle PQR$ and the volume V of the parallelepiped determined by $\overrightarrow{PQ}$, $\overrightarrow{PR}$, $\overrightarrow{PS}$.

A. $A = 9,\ V = 14$.

B. $A = 9,\ V = 7$.

C. $A = 9/2,\ V = 7$.

D. $A = 9/2,\ V = 14$.

E. $A = 9/2,\ V = 10$.

Record the correct answer to the following problems on the front page of this exam.

3. (8 points) Find the length L of the curve

$$\mathbf{r}(t) = \langle t^3/3, t^2, 2t \rangle, \quad (0 \leq t \leq 1).$$

A. $L = 5/2$.

B. $L = 5/3$.

C. $L = 7/2$.

D. $L = 7/3$.

E. $L = 9/2$.

4. (8 points) Find the curvature κ of the plane curve $y = x^5$ at the point $(1, 1)$.

A. $\kappa = \frac{5\sqrt{26}}{13}$.

B. $\kappa = \frac{5\sqrt{26}}{169}$.

C. $\kappa = \frac{4\sqrt{26}}{13}$.

D. $\kappa = \frac{4\sqrt{26}}{169}$.

E. $\kappa = \frac{3\sqrt{26}}{13}$.

Free Response Questions: Show your work!

5. (8 points) Find parametric equations for the line L passing through the point $(3,7,12)$ and perpendicular to the plane

$$x - y + 3z = 1.$$

6. (10 points) Find parametric equations for the line of intersection of the planes

$$3x + 2y - 5z = 1$$

and

$$3x - 2y - z = 0.$$

Free Response Questions: Show your work!

7. (10 points) A plane containing the points $P = (2, 1, 1), Q = (5, 7, -1), R = (5, 2, 1)$ has an equation of the form $2x + by + cz = d$. Find b, c, and d.

8. (10 points) Find parametric equations for the curve of intersection of the paraboloid $z = 4x^2 + 9y^2$ and the plane $4x + z = 3$.

Free Response Questions: Show your work!

9. (10 points) Identify the following surfaces and describe their locations:

(a) $x^2 + 3y^2 + 9z^2 + 6y + 2x = 50$

(b) $36x^2 - 9y^2 + 4z^2 = 36$.

10. (10 points) The acceleration vector at time t of a particle moving in space is given by

$$\mathbf{a}(t) = t^2\mathbf{i} + t\mathbf{j} + t^3\mathbf{k}.$$

Find the position vector $\mathbf{r}(1)$ of the particle at $t = 1$ if at $t = 0$ the particle is at the origin of $\mathbf{R}^3$ and its velocity is $\mathbf{v}(0) = 3\mathbf{i} + 4\mathbf{k}$.

Free Response Questions: Show your work!

11. (10 points) Compute the unit vectors $\mathbf{T}, \mathbf{N}$, and $\mathbf{B}$ for the curve

$$\mathbf{r}(t) = \langle \cos t, \sin t, \ln \cos t \rangle$$

at the point $(1, 0, 0)$. [You may need to use the identity $\sec^2 t = 1 + \tan^2 t$.]

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

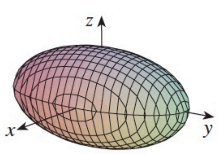
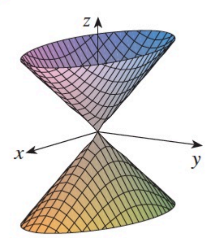
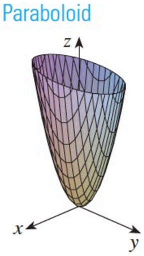
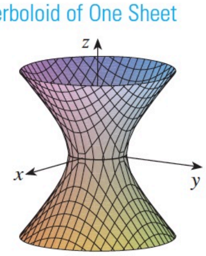
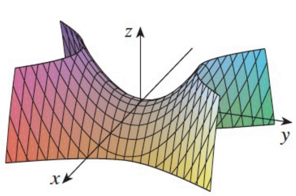
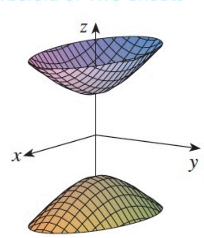
$$L = \int_a^b |\mathbf{r}'(t)|\, dt.$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}, \quad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}, \quad \mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t), \quad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

$$\kappa(x) = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}}.$$

$$x = (v_0 \cos\alpha)t, \qquad y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2.$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}, \qquad a_T = v' = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|}, \qquad a_N = \kappa v^2 = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}.$$

Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	Cone 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
Elliptic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.