MA 213 — Calculus III Fall 2016
Final Exam December 15, 2016

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:** Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:** Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		30
6		10
7		10
8		10
9		10
10		10
11		10
12		10
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. (6 points) Find the area of the parallelogram in 3-space with vertices $(-2, 1, 1)$, $(6, 4, 2)$, $(4, 3, 2)$, and $(12, 6, 3)$.

A. 0

B. 3

C. 9

D. 55

E. none of the above

Record the correct answer to the following problems on the front page of this exam.

2. (6 points) Which of the following parametric equations represent the tangent line to the curve

$$\mathbf{r}(t) = (t+1)\mathbf{i} + e^{-t}\mathbf{j} + (3t - t^2 + t^3)\mathbf{k}$$

at the point $(1, 1, 0)$?

A. $x = 1 + t, \quad y = 1 - t, \quad z = 3t$

B. $x = 2 + t, \quad y = 1 + t, \quad z = 3 + 3t$

C. $x = 1 + t, \quad y = 1 + t, \quad z = 3t$

D. $x = 1 + t, \quad y = 1 + t, \quad z = 3 + 3t$

E. $x = 1 - t, \quad y = -t, \quad z = 3 - 3t$

Record the correct answer to the following problems on the front page of this exam.

3. (6 points) Find the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+4}-2}.$$

A. 0

B. 2

C. 4

D. 6

E. 8

Record the correct answer to the following problems on the front page of this exam.

4. (6 points) The volume V of the solid below the plane $2x + y + z = 4$ and above the disk $x^2 + y^2 \leq 1$ in the xy-plane is

A. $V = 0$

B. $V = 2\pi$

C. $V = 4\pi$

D. $V = 6\pi$

E. $V = 8\pi$

Record the correct answer to the following problems on the front page of this exam.

5. (6 points) Use Green's Theorem to evaluate the integral

$$I = \oint_C (\sqrt{x^2+1} + y)\, dx + (\sqrt{y^2+1} - x)\, dy,$$

where C is the positively oriented circle $x^2 + y^2 = 3$.

A. $I = 3\pi$

B. $I = -3\pi$

C. $I = 6\pi$

D. $I = -6\pi$

E. $I = 9\pi$

Free Response Questions: Show your work!

6. (10 points) Find the point of intersection of the line through $(4, 1, 3)$ and $(-6, 5, 1)$ with the plane $x - y + 3z = 2$.

7. (10 points) Find an equation of the tangent plane to the surface $xy^2z^3 = 8$ at the point $(2, 2, 1)$. Write the answer in the form $x + by + cz = d$.

Free Response Questions: Show your work!

8. (10 points) Let $D = \{(x, y) \mid 4x^2 + y^2 \leq 8\}$ and let $f(x, y) = xy$. Find the absolute maximum and absolute minimum of $f(x, y)$ in D.

[Be sure to justify your work. Unsupported answers will not receive credit.]

Free Response Questions: Show your work!

9. (10 points) Find the volume of the part of the ball $\rho \leq 2$ between the cones $\phi = \pi/6$ and $\phi = \pi/3$.

10. (10 points) Verify that $\mathbf{F}(x, y) = \langle ye^x, e^x - e^y \rangle$ is a conservative vector field, find a potential function, and evaluate the integral

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where C is the curve with parametric equation

$$\mathbf{r}(t) = 2\cos^2(t)\,\mathbf{i} + 2\cos(t)\sin(t)\,\mathbf{j}, \quad \pi/4 \leq t \leq \pi/2.$$

Free Response Questions: Show your work!

11. (10 points) Evaluate

$$\int_C yz\, ds,$$

where C is the line segment from $(3, 1, 2)$ to $(1, 2, 5)$.

Free Response Questions: Show your work!

12. (10 points) Find the work done by the force field $\mathbf{F}(x, y) = x^3\mathbf{i} + y^3\mathbf{j}$ in moving an object along a path C from $P(1, 0)$ to $Q(2, 2)$.

$$\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\mathbf{a}\times\mathbf{b} = \det\begin{bmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{bmatrix}$$

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}.$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|},$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|},$$

$$\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t),$$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

$$x = (v_0\cos\alpha)t$$

$$y = (v_0\sin\alpha)t - \frac{1}{2}gt^2$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$$

$$a_T = v' = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$$

$$a_N = \kappa v^2 = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$m = \iint_R \rho(x,y)\,dA$$

$$M_y = \iint_R x\rho(x,y)\,dA$$

$$M_x = \iint_R y\rho(x,y)\,dA$$

$$I_x = \iint_R y^2\rho(x,y)\,dA$$

$$I_y = \iint_R x^2\rho(x,y)\,dA$$

$$I_0 = I_x + I_y = \iint_R (x^2+y^2)\rho(x,y)\,dA$$

$$(\overline{x},\overline{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

$$ds = |\mathbf{r}'(t)|\,dt$$

$$\mathrm{arclength}(C) = \int_C 1\,ds$$

$$\mathrm{area}(R) = \iint_R 1\,dA$$

$$\mathrm{volume}(D) = \iiint_D 1\,dV$$

$$\mathrm{area}(R) = \frac{1}{2}\oint_C x\,dy - y\,dx$$

$$\text{surface area}(S) = \iint_R \sqrt{f_x^2 + f_y^2 + 1}\,dA$$

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int \mathbf{F}\cdot\mathbf{T}\,ds$$