MA 213 — Calculus III Spring 2017
Exam 1 February 8, 2017

Name: _____

Section: _____

Last 4 digits of student ID #: _____

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:**
  Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:**
  Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		30
6		10
7		10
8		10
9		10
10		10
11		10
12		10
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. (6 points) Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors in 3-space and suppose $\mathbf{a} \times \mathbf{b} = \mathbf{0}$. Which of the following statements is correct?

A. $\mathbf{a} = \mathbf{b} = \mathbf{0}$

B. $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{0}$

C. $\mathbf{a}$ is perpendicular to $\mathbf{b}$

D. $\mathbf{a}$ and $\mathbf{b}$ are parallel

E. $\mathbf{a} + \mathbf{b} = \mathbf{0}$

2. (6 points) Which of the following vectors is parallel to the plane $3x - 5y + 7z = 10$?

A. $\mathbf{i} + \mathbf{k}$

B. $\mathbf{i} + 4\mathbf{j} - \mathbf{k}$

C. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

D. $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

E. $\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

Record the correct answer to the following problems on the front page of this exam.

3. (6 points) The radius of the sphere

$$x^2 + y^2 + z^2 - 2x + 4y - 4z = 16$$

is

A. 5

B. 4

C. 3

D. 2

E. 1

4. (6 points) If the angle between two unit vectors $\mathbf{u}$ and $\mathbf{v}$ is $2\pi/3$, then

A. $\mathbf{u} \cdot \mathbf{v} = 1$

B. $\mathbf{u} \cdot \mathbf{v} = 1/2$

C. $\mathbf{u} \cdot \mathbf{v} = 0$

D. $\mathbf{u} \cdot \mathbf{v} = -1/2$

E. $\mathbf{u} \cdot \mathbf{v} = -1$

Record the correct answer to the following problems on the front page of this exam.

5. (6 points) A vector function $\mathbf{r}(t)$ satisfies $|\mathbf{r}(t)| = t$ for all t. Which of the following is true?

A. $\mathbf{r}(t) \cdot \mathbf{r}'(t) = t^2$ for all t

B. $\mathbf{r}(t) \cdot \mathbf{r}'(t) = t$ for all t

C. $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 1$ for all t

D. $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 2t$ for all t

E. $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$ for all t

6. (10 points) Find parametric equations for the tangent line to the curve

$$\mathbf{r}(t) = \langle t^2, t^3, t \rangle$$

at the point $(1, -1, -1)$.

7. (10 points) Write an equation for the plane passing through $(1, 1, 2)$ and parallel to the y-axis and also to the line

$$x = 1 + 2t, \quad y = 3 - t, \quad z = 4 - 5t.$$

Write the answer in the form $z = ax + by + c$.

8. (10 points) Find the curvature of the graph of $y = 2\sin x$ at the point $(\pi/6, 1)$.

9. (10 points) A ball is thrown at an angle 45° to the ground. If the ball lands 90 m away,

(a) Find the time t when the ball hits the ground;

(b) Find the initial speed of the ball.

10. (10 points) Find $\text{proj}_{\mathbf{b}}\,\mathbf{a}$ of the vector $\mathbf{a} = \langle 3, -1, -1\rangle$ along the vector $\mathbf{b} = \langle 3, -4, 12\rangle$.

11. (10 points) Find the unit tangent vector $\mathbf{T}$ and the unit normal vector $\mathbf{N}$ for the curve

$$\mathbf{r}(t) = \langle 2t, 3\cos t, 3\sin t \rangle$$

at the point where $t = 1$.

12. (10 points) Find an equation for the plane through $(2, 0, 0)$, $(0, 1, 1)$ and $(0, 0, 3)$. Write the answer in the form $ax + by + cz = 6$.

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

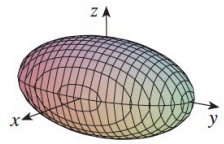
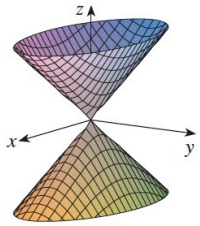
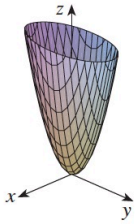
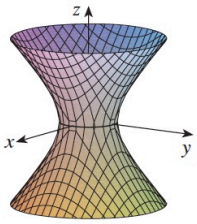
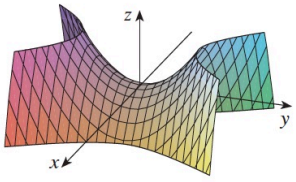
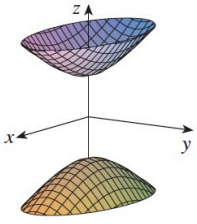
$$L = \int_a^b |\mathbf{r}'(t)|\, dt.$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}, \quad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}, \quad \mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t), \quad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

$$\kappa(x) = \frac{|f''(x)|}{(1 + (f'(x))^2)^{3/2}}.$$

$$x = (v_0 \cos\alpha)t, \qquad y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2.$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}, \qquad a_T = v' = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|}, \qquad a_N = \kappa v^2 = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}.$$

Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	Cone 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
Elliptic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.