

Name: _____

Section: _____

Last 4 digits of student ID #: _____

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:**
Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:**
Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		30
6		10
7		10
8		10
9		10
10		10
11		10
12		10
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. (6 points) If $f(x, y) = (x + y^2)^3$, compute $f_{xy}(2, 1)$:

- A. 9
- B. 18
- C. 36
- D. 48
- E. 72

2. (6 points) Suppose $f(x, y)$ is a differentiable function,

$$x(r, s, t) = e^r + 2e^s + 3e^t, \quad y(r, s, t) = e^{r+s+t}.$$

and

$$g(r, s, t) = f(x(r, s, t), y(r, s, t)).$$

Use the following table of values to calculate $g_s(0, 0, 0)$:

	f	g	f_x	f_y
$(0, 0)$	3	5	-1	3
$(1, 1)$	5	4	2	5
$(3, 1)$	4	1	4	-1
$(6, 1)$	2	7	3	2

- A. 0
- B. 2
- C. 4
- D. 6
- E. 8

Record the correct answer to the following problems on the front page of this exam.

3. (6 points) Suppose $(0, 0)$ is a critical point of a function $f(x, y)$ with continuous second derivatives and

$$f_{xx}(0, 0) = 4, \quad f_{xy}(0, 0) = -6, \quad f_{yy}(0, 0) = 9,$$

What does the second derivative test say about f at $(0, 0)$?

- A. f has a local maximum at $(0, 0)$
- B. f has an absolute maximum at $(0, 0)$
- C. f has a local minimum at $(0, 0)$
- D. f has a saddle point at $(0, 0)$
- E. The second derivative test is inconclusive

Record the correct answer to the following problems on the front page of this exam.

4. (6 points) The gradient of $f(x, y)$ at $(1, -1)$ is $\langle 7, 1 \rangle$. Find the rate of change of $f(x, y)$ at $(1, -1)$ in the direction of $4\mathbf{i} - 3\mathbf{j}$.
- A. 0
 - B. 5
 - C. 25
 - D. 31
 - E. -31

Record the correct answer to the following problems on the front page of this exam.

5. (6 points) Find the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2 - x^2 - y^2}{x^2 + y^2}.$$

- A. -1
- B. 0
- C. 1
- D. ∞
- E. Does not exist

Free Response Questions: Show your work!

- 6.** (10 points) Find all the critical points of $f(x, y) = xy + e^{-xy}$.

Free Response Questions: Show your work!

7. (10 points) Use Lagrange multipliers to find absolute maximum and minimum values of $f(x, y, z) = xyz$ on the sphere of radius $\sqrt{3}$ centered at the origin. List all the points where $f(x, y, z)$ reaches its maximum and minimum values. (No credit will be given if another method is used. Your answer must be justified.)

Free Response Questions: Show your work!

8. (10 points) Let $f(x, y) = e^{x/y}$. Compute the linearization of f at $(\ln 2, 1/2)$ following these steps:

(a) Compute $f(\ln 2, 1/2)$.

(b) Compute $f_x(\ln 2, 1/2)$.

(c) Compute $f_y(\ln 2, 1/2)$.

(d) Compute the linearization of f at $(\ln 2, 1/2)$.

Free Response Questions: Show your work!

9. (10 points) Use implicit differentiation to compute $\partial z/\partial x$ at $(-\pi, 1, \pi)$ if

$$\sin(y^2 z) = x^3 + z^3.$$

Free Response Questions: Show your work!

10. (10 points) Use gradients to find an equation for the tangent plane to the surface $x^3 + y^3 + z^3 - 6xyz = 7$ at the point $(1, -1, 1)$. Write the equation in the form $3x + by + cz = d$.

Free Response Questions: Show your work!

- 11.** (10 points) Find the absolute minimum and maximum values of $f(x, y) = xy - x - y$ on the triangle bounded by the x -axis, the y -axis, and the line $2x + 3y = 6$.

Free Response Questions: Show your work!

- 12.** (10 points) Use the linear approximation of $f(x, y) = x^2 + xy + y^2$ at $(3, 2)$ to approximate $f(2.9, 2.2)$. (No credit will be given for using a calculator estimate.)