MA 213 — Calculus III Spring 2017
Exam 3 April 12, 2017

Name: ______________________________

Section: ____________________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:**
Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:**
Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		30
6		10
7		10
8		10
9		10
10		10
11		10
12		10
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. (6 points) Which of the following surfaces is represented in spherical coordinates by the equation $\rho = \cos\phi$?

A. A sphere

B. A plane

C. A cone

D. A cylinder

E. None of the above

2. (6 points) The rectangular coordinates of a point P in 3-space are $(\sqrt{3}, -1, 2\sqrt{3})$. Find the spherical coordinates (ρ, θ, ϕ) of P.

A. $(4, -\pi/6, \pi/3)$

B. $(4, \pi/6, \pi/3)$

C. $(4, \pi/3, \pi/6)$

D. $(4, -\pi/6, \pi/6)$

E. $(4, 2\pi/3, \pi/6)$

Record the correct answer to the following problems on the front page of this exam.

3. (6 points) The average value of $f(x, y, z) = xy^2z^3$ over the ball

$$B = \left\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 2\right\}.$$

is

A. -2

B. -1

C. 0

D. 1

E. 2

4. (6 points) Which of the following iterated integrals represents the volume enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1$?

A. $\displaystyle\int_0^{2\pi}\int_0^1\int_0^1 r\,dz\,dr\,d\theta$

B. $\displaystyle\int_0^{2\pi}\int_0^1\int_r^1 r\,dz\,dr\,d\theta$

C. $\displaystyle\int_0^{2\pi}\int_r^1\int_0^1 r\,dr\,dz\,d\theta$

D. $\displaystyle\int_0^{2\pi}\int_0^1\int_r^1 dz\,dr\,d\theta$

E. $\displaystyle\int_0^{\pi}\int_0^1\int_0^1 r\,dz\,dr\,d\theta$

Record the correct answer to the following problems on the front page of this exam.

5. (6 points) If $f(x, y)$ is a continuous function in the disk $D = \{(x, y) \mid x^2 + y^2 \leq 1\}$, and $1 \leq f(x, y) \leq 2$ in D, which of the following inequalities is *false*?

A. $\displaystyle\iint_D f(x, y)\, dA \geq 1$

B. $\displaystyle\iint_D f(x, y)\, dA \geq \pi$

C. $\displaystyle\iint_D f(x, y)\, dA \leq 8$

D. $\displaystyle\iint_D f(x, y)^2\, dA \geq 16$

E. $\displaystyle\iint_D f(x, y)^2\, dA \leq 15$

Free Response Questions: Show your work!

6. (10 points) Let E be the region bounded by the planes $x = 0$, $z = 0$, $y = 2x$, and $x + y + z = 1$. Write

$$\iiint_D f(x, y, z)\, dV$$

as an iterated integral

$$\int \int \int f(x, y, z)\, dz\, dy\, dx$$

(determine the correct limits of integration).

Free Response Questions: Show your work!

7. (10 points) Let D be the triangular region in the plane with vertices $(-1, 3), (1, 3), (0, 0)$. Find the centroid of D.

Free Response Questions: Show your work!

---

8. (10 points) Let E be the region below the cone $z = 1 - \sqrt{x^2 + y^2}$ and above the xy-plane. Write an iterated integral for computing the volume of E using cylindrical coordinates. Do not evaluate the integral.

Free Response Questions: Show your work!

9. (10 points) Change the order of integration in

$$\int_0^2 \int_0^{x^2} f(x,y)\, dy\, dx.$$

Do not evaluate the integral.

Free Response Questions: Show your work!

10. (10 points) Let E be the region between the spheres

$$x^2 + y^2 + z^2 = 2z$$

and

$$x^2 + y^2 + z^2 = 4z.$$

Write an iterated integral using spherical coordinates for computing

$$\iiint_E f(x, y, z)\, dV.$$

Do not evaluate the integral.

Free Response Questions: Show your work!

11. (10 points) Knowing that the volume of a sphere of radius R is $4\pi R^3/3$, evaluate

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{\sqrt{4-x^2-y^2}} 3\, dz\, dx\, dy.$$

Free Response Questions: Show your work!

12. (10 points) Consider a disk D of radius R centered at the origin in the xy-plane. Find the average distance from the center of the disk to a point of D (i.e. the average value of the function $f(x, y) = \sqrt{x^2 + y^2}$ over D).

$$f_{\text{ave}} = \frac{1}{\text{Area}(R)} \iint_R f(x,y)\,dA \qquad\qquad f_{\text{ave}} = \frac{1}{\text{Vol}(D)} \iiint_D f(x,y,z)\,dV$$

$$m = \iint_R \rho(x,y)\,dA$$

$$M_y = \iint_R x\rho(x,y)\,dA \qquad M_x = \iint_R y\rho(x,y)\,dA$$

$$(\overline{x}, \overline{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

$$\text{Surface Area}(S) = \iint_R \sqrt{(f_x(x,y))^2 + (f_y(x,y))^2 + 1}\,dA$$

If

$$m \le f(x,y) \le M$$

for $(x,y) \in D$, then

$$m\,\text{Area}(D) \le \iint_D f(x,y)\,dA \le M\,\text{Area}(D)$$

$$m = \iiint_D \rho(x,y,z)\,dV$$

$$M_{yz} = \iiint_D x\rho(x,y,z)\,dV \qquad M_{xz} = \iiint_D y\rho(x,y,z)\,dV \qquad M_{xy} = \iiint_D z\rho(x,y,z)\,dV$$

$$(\overline{x}, \overline{y}, \overline{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m}\right)$$

$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$