Name: ______________________ Section and/or TA: ______________

Last Four Digits of Student ID: ________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used except for a one-page sheet of formulas and facts. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response questions. Unsupported answers on free response questioins will receive *no credit*.

Multiple Choice Questions

1	A	B	C	D	E	**6**	A	B	C	D	E
2	A	B	C	D	E	**7**	A	B	C	D	E
3	A	B	C	D	E	**8**	A	B	C	D	E
4	A	B	C	D	E	**9**	A	B	C	D	E
5	A	B	C	D	E	**10**	A	B	C	D	E

SCORE

Multiple Choice	11	12	13	14	Total Score
60	10	10	10	10	100

Multiple Choice Questions

1. (6 points) Which of the following points in $\mathbf{R}^3$ lies on the line through $P(1,2,1)$ and $Q(7,0,3)$?

   A. $(4,2,2)$

   B. $(-1,4,1)$

   C. $(2,3,0)$

   D. $(-2,3,0)$

   E. None of the above

2. (6 points) If $\mathbf{a} = -2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$, then the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$ is

   A. $5/7$

   B. $16/21$

   C. $3/4$

   D. $-16/21$

   E. $-5/7$

3. (6 points) Consider the points $P(-1,2,1)$ and $Q(1,3,-1)$. The vector $\mathbf{v}$ of magnitude 3 in the direction of $\overrightarrow{PQ}$ is

   A. $\langle 2,1,-2 \rangle$

   B. $\langle 2,-1,2 \rangle$

   C. $\langle -2/3,1/3,2/3 \rangle$

   D. $\langle 2/3,1/3,2/3 \rangle$

   E. $\langle 2,1,2 \rangle$

4. (6 points) Consider the vectors $\mathbf{a} = \langle 1,2,3 \rangle$, $\mathbf{b} = \langle 1,-1,-2 \rangle$, and $\mathbf{c} = \langle 2,1,4 \rangle$. The scalar triple product $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ is

   A. 3

   B. 6

   C. 9

   D. -9

   E. 12

5. (6 points) If the symmetric equations of a line L are

$$\frac{x+1}{3} = \frac{y-2}{2} = z+1,$$

then the parametric equations of L are

A. $x = 1+3t,\ y = -2+2t,\ z = 1+t$

B. $x = -1-3t,\ y = 2-2t,\ z = -1+6t$

C. $x = -2+3t,\ y = 3+2t,\ z = -6+6t$

D. $x = 1+3t,\ y = 2+2t,\ z = 1+t$

E. $x = -1+3t,\ y = 2+2t,\ z = -1+t$

6. (6 points) Consider the sphere S with equation $x^2+y^2+z^2-2x+4y = 4$. Which of the following equations represents a sphere with the same center as S but with a radius twice the radius of S?

A. $x^2+y^2+z^2-2x+4y = 36$

B. $x^2+y^2+z^2-2x+4y = 32$

C. $x^2+y^2+z^2-2x+4y = 31$

D. $x^2+y^2+z^2-2x+4y = 16$

E. $x^2+y^2+z^2-2x+4y = 8$

7. (6 points) Find **all** the points on the helix

$$\mathbf{r}(t) = \langle 4\cos(\pi t), 4\sin(\pi t), 9t\rangle$$

whose distance from the origin is 5.

A. $\langle 2, 2\sqrt{3}, 3\rangle$

B. $\langle 2\sqrt{3}, 2, 3\rangle$ and $\langle 2\sqrt{3}, -2, -3\rangle$

C. $\langle 2\sqrt{3}, -2, 3\rangle$ and $\langle 2\sqrt{3}, 2, -3\rangle$

D. $\langle 2, 2\sqrt{3}, -3\rangle$ and $\langle 2, -2\sqrt{3}, 3\rangle$

E. $\langle 2, 2\sqrt{3}, 3\rangle$ **and** $\langle 2, -2\sqrt{3}, -3\rangle$

8. (6 points) Consider the planes

$$\begin{aligned} P_1 &: \; x + y + 2z = 0 \\ P_2 &: \; 5x + 3y - 4z = 1 \end{aligned}$$

and the line

$$L: \quad x = 5t, \quad y = 1 - 7t, \quad z = 3 + t.$$

Choose the correct statement:

A. L is perpendicular to both P_1 and P_2

B. L is perpendicular to P_1 and parallel to P_2

C. L is perpendicular to P_2 and parallel to P_1

D. L is parallel to both P_1 and P_2

E. None of the above

9. (6 points) Consider the curve

$$C: \quad \mathbf{r}(t) = \langle t^2, t^2 + 1, t^3 - 1 \rangle.$$

Which of the following lines is tangent to C at the point $(1, 2, -2)$?

A. $x = 1 + 2t, y = 2 - 2t, z = -2 + 3t$

B. $x = 1 - 2t, y = -2 - 2t, z = 2 + 3t$

C. $x = 1 - 2t, y = 2 - 2t, z = -2 + 3t$

D. $x = 1 + 2t, y = 2 - 2t, z = -2 - 3t$

E. $x = 1 - 2t, y = 2 + 2t, z = -2 + 3t$

10. (6 points) Consider the triangle with vertices $P(5, -1, 4)$, $Q(3, 1, 4)$, $R(3, -1, 6)$. Which one of the following statements is correct?

A. ΔPQR is an equilateral triangle

B. ΔPQR is an isosceles triangle

C. ΔPQR is a right triangle, but not isosceles

D. ΔPQR has no equal sides

E. None of the above

Free Response Questions

11. (10 points) The plane P contains the line

$$L_1: \quad x = 1 - t, \quad y = 3 + t, \quad z = 2t$$

and is parallel to the line

$$L_2: \quad x = 4 - 3t, \quad y = 3 + 2t, \quad z = 7 + t.$$

(a) (5 points) Find an equation for the plane P. Write the equation in the form

$$ax + by + cz + d = 0.$$

Solution: The normal vector for P must be perpendicular to the direction vectors of L_1 and L_2. Thus we take

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 2 \\ -3 & 2 & 1 \end{vmatrix} = \langle -3, -5, 1 \rangle.$$

Thus we can write the equation of P as

$$-3x - 5y + z + d = 0.$$

To find d, we note that the point $(1, 3, 0)$ is on L_1 and therefore on P. Thus

$$-3 \cdot 1 - 5 \cdot 3 + d = 0,$$

which gives $d = 18$ and the equation of P is

$$-3x - 5y + z + 18 = 0.$$

(b) (5 points) Find the distance between L_2 and P.

Solution: Since the point $(4, 3, 7)$ is on L_2, the distance from L_2 to P is

$$\frac{|-3 \cdot 4 - 5 \cdot 3 + 7 + 18|}{\sqrt{3^2 + 5^2 + 1^2}} = \frac{2}{\sqrt{35}}.$$

12. (10 points) Knowing that the velocity of a particle moving in space is

$$\mathbf{v}(t) = \langle t, \cos(\pi t), 3t^2 \rangle \qquad (t \geq 0)$$

and that at $t = 0$, the particle is at the origin, find the position of the particle at $t = 2$.

Solution: If $\mathbf{r}(t)$ describes the motion of the particle, we have $\mathbf{r}'(t) = \mathbf{v}(t)$. Thus

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt = \left\langle \frac{t^2}{2}, \frac{\sin(\pi t)}{\pi}, t^3 \right\rangle + \mathbf{c}.$$

Since $\mathbf{r}(0) = \mathbf{0}$, we see that $\mathbf{c} = \mathbf{0}$. Thus

$$\mathbf{r}(t) = \left\langle \frac{t^2}{2}, \frac{\sin(\pi t)}{\pi}, t^3 \right\rangle$$

and

$$\mathbf{r}(2) = \langle 2, 0, 8 \rangle .$$

13. (10 points) Find a vector function $\mathbf{r}(t)$ parametrizing the curve of intersection of the cylinder $(x-1)^2 + y^2 = 1$ and the paraboloid $z = x^2 + y^2$.

Solution: The projection of the curve onto the xy-plane the circle of radius 1 centered at $(1, 0)$. Thus we can take $x = \cos t + 1$, $y = \sin t$. Then

$$z = (\cos t + 1)^2 + (\sin t)^2 = 2 + 2\cos t$$

and

$$\mathbf{r}(t) = \langle \cos t + 1, \sin t, 2 + 2\cos t \rangle.$$

14. (10 points) Find the length of the curve described by the vector function

$$\mathbf{r}(t) = \langle 1, 3t^2, 2t^3 \rangle \qquad (0 \le t \le 1).$$

Solution: We have

$$\mathbf{r}'(t) = \langle 0, 6t, 6t^2 \rangle.$$

Thus the length of the curve is

$$\begin{aligned} \int_0^1 |\mathbf{r}'(t)|\, dt &= \int_0^1 \sqrt{36t^2 + 36t^4}\, dt = \int_0^1 6t\sqrt{1+t^2}\, dt \\ &= 3\int_1^2 \sqrt{u}\, du = \left[2u^{3/2}\right]_1^2 = 2(\sqrt{8} - 1), \end{aligned}$$

where we used the substitution $u = 1 + x^2,\ du = 2x\, dx$.