Name: ______________________ Section and/or TA: ______________

Last Four Digits of Student ID: ________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 4 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response questions. Unsupported answers on free response questioins will receive *no credit*.

Multiple Choice Questions

1	A	B	C	D	E	**6**	A	B	C	D	E
2	A	B	C	D	E	**7**	A	B	C	D	E
3	A	B	C	D	E	**8**	A	B	C	D	E
4	A	B	C	D	E	**9**	A	B	C	D	E
5	A	B	C	D	E	**10**	A	B	C	D	E

SCORE

Multiple Choice	11	12	13	14	Total Score
60	12	8	10	10	100

Multiple Choice Questions

1. (6 points) Which of the following points in $\mathbf{R}^2$ does **not** belong to the domain of

$$f(x,y) = \ln[(4-x^2)(y^2-9)]?$$

   A. $(-3,-1)$
   B. $(1,-4)$
   C. $(-3,-2)$
   D. $(-1,2)$
   E. $(3,1)$

2. (6 points) One of the following limits does not exist. Which one?

   A. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x}{x+y}$
   B. $\lim\limits_{(x,y)\to(2,1)} e^{\sqrt{2x-y}}$
   C. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4-y^4}{x^2+y^2}$
   D. $\lim\limits_{(x,y)\to(0,0)} \ln[(x-1)(y-1)]$
   E. $\lim\limits_{(x,y)\to(2,1)} \dfrac{x^2y+xy^2}{x^2-y^2}$

3. (6 points) Compute $f_x(3,4)$, if $f(x,y) = \ln(x^2 - \sqrt{x^2+y^2})$:

   A. $1/4$
   B. $27/14$
   C. $33/14$
   D. $33/20$
   E. $27/20$

4. (6 points) Find an equation for the tangent plane to the surface $z = e^{y-x}$ at the point $(2, 2, 1)$:

   A. $x + y - z = 1$

   B. $x - y - z = 1$

   C. $x - y + z = 1$

   D. $x + y + x = 1$

   E. $x - y + z = -1$

5. (6 points) If $x = \cos t$, $y = \sin t$, what is the derivative of $f(x, y) = x^2y^3$ with respect to t, when $t = \pi/6$?

   A. 0

   B. $(1/8)\sqrt{3}$

   C. $(5/32)\sqrt{3}$

   D. $(3/16)\sqrt{3}$

   E. $(7/32)\sqrt{3}$

6. (6 points) If $e^{xz} = xy$, use implicit differentiation to compute $\dfrac{\partial z}{\partial x}$ at the point $(1, 1, 0)$

   A. 0

   B. 1

   C. -1

   D. 2

   E. -2

7. (6 points) The derivative of $f(x, y) = xy(x + y)$ and the point $(1, 2)$ in the direction of $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$

   A. $44/5$

   B. 9

   C. $46/5$

   D. $47/5$

   E. $48/5$

8. (6 points) If

$$f_x(0,0) = f_y(0,0) = f_x(1,1) = f_y(1,1) = 0$$

and

$$f_{xx}(0,0) = 2, \quad f_{yy}(0,0) = 5, \quad f_{xy}(0,0) = f_{yx}(0,0) = 3,$$
$$f_{xx}(1,1) = 2, \quad f_{yy}(1,1) = 4, \quad f_{xy}(1,1) = f_{yx}(1,1) = 3,$$

then $f(x,y)$ has

A. A local minimum at $(0,0)$ and a local maximum at $(1,1)$

B. A local maximum at $(0,0)$ and a local minimum at $(1,1)$

C. A saddle point at $(0,0)$ and a local maximum at $(1,1)$

D. A local maximum at $(0,0)$ and a saddle point at $(1,1)$

E. A local minimum at $(0,0)$ and a saddle point at $(1,1)$

9. (6 points) Evaluate

$$\iint_R \left(\frac{y^2+1}{x^2+1}\right) dA, \quad \text{where} \quad R = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1\}.$$

A. 0

B. $\pi/4$

C. $\pi/3$

D. $\pi/2$

E. π

10. (6 points) Evaluate

$$\iint_D x^2\, dA, \quad \text{where } D \text{ is bounded by } y = 1 - x^2 \text{ and } y = 0.$$

A. $1/15$

B. $2/15$

C. $1/5$

D. $4/15$

E. $1/3$

Free Response Questions

11. (12 points) Use Lagrange multipliers to fine the absolute maximum and absolute minimum values of $f(x,y) = x^4 + y^4$ on the disk $x^2 + y^2 \leq 4$ and the points where these values are reached. **Note**: you must use Lagrange multipliers to receive full credit for this problem.

(a) (2 points) Find the critical point(s) of $f(x,y)$ inside the disk.

Solution: We have $\nabla f(x,y) = \langle 4x^3, 4y^3 \rangle$. Thus the only critical point of $f(x,y)$ inside the disk is $(0,0)$, where $f(0,0) = 0$.

(b) (4 points) Turning to the boundary circle, consider first the points where either x or y is zero.

Solution: The boundary of the disk is the circle $x^2 + y^2 = 4$. If $x = 0$, we have $y = \pm 2$. Similarly, if $y = 0$, we have $x = \pm 2$. This gives 4 points

$$(0,2), \quad (0,-2), \quad (2,0), \quad (-2,0).$$

The value of $f(x,y)$ at all four of these points is 16.

(c) (4 points) Consider the points of the boundary where neither x nor y is zero (use Lagrange multipliers).

Solution: Since the constraint is $g(x,y) = x^2 + y^2 = 4$ and $\nabla g(x,y) = \langle 2x, 2y \rangle$, we need to solve for x and y the three equations

$$4x^3 = 2\lambda x, \quad 4y^3 = 2\lambda y, \quad x^2 + y^2 = 4$$

assuming that $x \neq 0$ and $y \neq 0$. From the first two equations we have

$$\lambda = 2x^2 = 2y^2$$

and since $x^2 + y^2 = 4$, we get $x^2 = y^2 = 2$, i.e., $x = \pm\sqrt{2}$ and $y = \pm\sqrt{2}$. This gives four points

$$(\sqrt{2}, \sqrt{2}), \quad (\sqrt{2}, -\sqrt{2}), \quad (-\sqrt{2}, \sqrt{2}), \quad (-\sqrt{2}, -\sqrt{2}),$$

and $f(x,y)$ has value 8 at each of these points.

(d) (2 points) Compare the values of $f(x,y)$ at the points found in (a), (b), and (c), and determine the points where the absolute maximum and absolute minimum are reached.

Solution: Comparing now the values of $f(x,y)$ at the 9 points found in (a), (b), (c), we conclude that $f(x,y)$ has an absolute maximum 16 at

$$(0,2), \quad (0,-2), \quad (2,0), \quad (-2,0)$$

and an absolute minimum 0 at $(0,0)$.

12. (8 points) Let $f(x,y) = x^2 e^y$

(a) (5 points) Find the linearization $L(x,y)$ of $f(x,y)$ at the point $(1,0)$.

Solution: We have

$$f_x(x,y) = 2xe^y, \quad f_y(x,y) = x^2 e^y$$

and therefore

$$f(1,0) = 1, \quad f_x(1,0) = 2, \quad f_y(1,0) = 1.$$

Thus

$$L(x,y) = 1 + 2(x-1) + 1(y-0) = 2x + y - 1.$$

(b) (3 points) Use the linearization found in (a) to approximate $f(0.98, 0.3)$.

Solution: We have

$$f(0.98, 0.3) \approx L(0.98, 0.3) = 1 + 2(-0.02) + 1(0.3) = 1.26.$$

13. (10 points) Let $R = \{(x, y) \mid 0 \le x \le 2, 0 \le y \le (x+1)^2\}$,

(a) (4 points) Find the area of R.

Solution: We have

$$\text{Area}(R) = \int_0^2 (x+1)^2 \, dA = \left[\frac{(x+1)^3}{3}\right]_{x=0}^{x=2} = \frac{7}{3}.$$

(b) (5 points) Evaluate

$$\iint_R y \, dA.$$

Solution: Since R is clearly a region of type I, we have

$$\iint_R y \, dA = \int_0^2 \int_0^{(x+1)^2} y \, dy \, dx = \int_0^2 \frac{(x+1)^4}{2} \, dx = \left[\frac{(x+1)^5}{10}\right]_{x=0}^{x=2} = \frac{121}{5}.$$

(c) (1 point) Find the average value of $f(x, y) = y$ over R.

Solution: We have

$$f_{\text{ave}} = \frac{1}{\text{Area}(R)} \iint_R y \, dA = \frac{3}{7} \cdot \frac{121}{5} = \frac{363}{35}.$$

14. (10 points) Find a vector equation for the normal line to the paraboloid $x^2 + y^2 - z = 0$ at the point $(2, -2, 1)$.

Solution: The gradient of $f(x, y, z) = x^2 + y^2 - z$ is

$$\nabla f(x, y, z) = \langle 2x, 2y, -1 \rangle.$$

Thus we can take $\nabla f(2, -2, 1) = \langle 4, -4, -1 \rangle$ as the direction vector $\mathbf{n}$ of the normal line andwe can take

$$\mathbf{r}(t) = \langle 2, -2, 1 \rangle + t\langle 4, -4, -1 \rangle = \langle 2 + 4t, -2 - 4t, 1 - t \rangle$$

as a vector equation of the normal line.