Name: ____________________ Section and/or TA: ____________

1. (4 points) Consider the surfaces S_1 and S_2 in $\mathbf{R}^3$ given by the equations

$$\begin{aligned}(S_1) \qquad & x^2+y^2+z^2-4x-6z+4=0,\\ (S_2) \qquad & x^2+y^2+z^2-6y+6z+2=0.\end{aligned}$$

(a) (2 points) Show that S_1 and S_2 are spheres and find the coordinates of their centers C_1 and C_2 and their radii R_1 and R_2.

Solution: Completing the squares, we rewrite the two equations as

$$\begin{aligned}(S_1) \qquad & (x-2)^2+y^2+(z-3)^2=9,\\ (S_2) \qquad & x^2+(y-3)^2+(z+3)^2=16.\end{aligned}$$

These are equations of two spheres and

$$C_1=(2,0,3), \quad C_2=(0,3,-3), \qquad R_1=3, \quad R_2=4.$$

(b) (1 point) Find the distance D between C_1 and C_2.

Solution: We have

$$D=\sqrt{(0-2)^2+(3-0)^2+(-3-3)^2}=\sqrt{49}=7.$$

(c) (1 point) Check the correct answer (no explanation necessary):

☐ S_1 and S_2 have no points in common

■ S_1 and S_2 have one point in common (i.e. are tangent)

☐ S_1 and S_2 have infinitely many points in common

Solution: Since $D=R_1+R_2$, the two spheres are tangent to each other and have a unique common point.