Bootstrap and Empirical Likelihood

Given a sample $X_1, X_2, ..., X_n$.

The empirical distribution $\hat{F}(t)$, and statistic $\hat{\theta} = \hat{\theta}(X_1, ..., X_n)$. (a pair)

The true distribution $F_0(t)$ and the true value of the parameter θ_0. (another pair)

Nonparametric Bootstrap method:

Fix the distribution at the empirical distribution $\hat{F}(t)$ and watch how the statistic change from $\hat{\theta}(X_1, ..., X_n)$ to $\hat{\theta}(Y_1, ..., Y_n)$. where Y_i is a sample from the empirical distribution $\hat{F}(t)$.

For example, the distribution of $[\hat{\theta}(Y_1, ...Y_n) - \hat{\theta}(X_1, ...X_n)]$ may be of interest.

Simulation Method:

is to fix the disrtibution at $F_0(t)$ and look at (the distribution of) the distance $[\hat{\theta}(X_1, ..., X_n) - \theta_0]$ ...... but $F_0(t)$ is (almost always) unknown.

Empirical Likelihood method:

Fix the $\hat{\theta}(X_1, ..., X_n)$ at true value (under H_o) θ_0 and see how much the empirical distribution got tilted in order to achieve this.

The distance between the empirical distribution $\hat{F}(t)$ and the tilted distribution F_λ is of interest. The distance is measured by log likelihood ratio. (or by (tilting parameter) λ)

There is a chi square reference to measure this log likelihood distance. (how much distance is reasonable, how much is too large...)