AL-JABAR

A Game of Strategy

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for the Celebration of Mind

AL-JABAR – Concepts

The game of *Al-Jabar* is based on concepts of color-mixing familiar to most of us from childhood, and on ideas from Abstract Algebra, a branch of higher mathematics. Once you are familiar with the rules of the game, your intuitive notions of color lead to interesting and often counter-intuitive color combinations.

Because *Al-Jabar* requires some preliminary understanding of the color-mixing mechanic before beginning to play the game, the rules are organized somewhat differently than most rulebooks. This first section details the "arithmetic" of adding (the mathematical equivalent of mixing) colors. While the mathematics involved uses some elements of group theory, a foundational topic in Abstract Algebra, understanding this "arithmetic" is not difficult and requires no mathematical background. The second section explains the process of play, and how this arithmetic of colors is used in the game.

Gameplay consists of manipulating game pieces in the three primary colors red, blue and yellow, which we denote in writing by R, B, and Y respectively; the three secondary colors green, orange and purple, which we denote by G, O, and P; the color white, denoted by W; and clear pieces, denoted by C, which are considered to be "empty" as they do not contain any color.

We refer to a game piece by its color, e.g. a red piece is referred to as "red," or R.

We use the symbol "+" to denote a combination, or grouping together, of colored game pieces; and call such a combination a "sum of colors."

We use the symbol "=" to mean that two sets of pieces are equal, or interchangeable, according to the rules of the game; that is, the sets have the same sum.

The order of a set of colors does not affect its sum; the pieces can be placed however you like.

Keep in mind as you read on, that these equations just stand for clusters of pieces. Try to see pictures of colorful pieces, not black-and-white symbols, in your mind. Try to imagine a red piece when you read "R," a blue and a green piece when you read "B + G," and so on. In mathematics, symbols are usually just a black-and-white way to write something much prettier.

Here are four of the defining rules in *Al-Jabar*, from which the entire game follows:

$$P = R + B$$

indicates that purple is the sum of red and blue, i.e. a red and a blue may be exchanged for a purple during gameplay, and vice versa;

$$O = R + Y$$

indicates that orange is the sum of red and yellow;

$$G = B + Y$$

indicates that green is the sum of blue and yellow; and a less obvious rule

$$W = R + B + Y$$

indicates that white is the sum of red, blue and yellow, which reminds us of the fact that white light contains all the colors of the spectrum—in fact, we see in the above equation that the three secondary colors R + B, R + Y and B + Y are also contained in the sum W.

In addition, there are two rules related to the clear pieces. Here we use red as an example color, but the same rules apply to every color, including clear itself:

$$R + C = R$$

indicates that a sum of colors is not changed by adding or removing a clear; and a special rule

$$R + R = C$$

indicates that two pieces of the same color (referred to as a "double") are interchangeable with a clear in gameplay.

It follows from the above two rules that if we have a sum containing a double, like R + B + B, then

$$R + B + B = R + C$$

as the two blues are equal to a clear. But R + C = R so we find that

$$R + B + B = R,$$

which indicates that a sum of colors is not changed by adding or removing a double—the doubles are effectively "cancelled" from the sum.

It also follows from these rules, if we replace R and B with C in the above equations, that

$$C + C = C$$

and

$$C + C + C = C.$$

We note that all groups of pieces having the same sum are interchangeable in *Al-Jabar*.
For instance,

$$Y + O = Y + R + Y = R + C = R,$$

as orange may be replaced by R + Y, and then the double Y + Y may be cancelled from the sum.
But it is also true that

$$B + P = B + R + B = R + C = R,$$

and even

$$G + W = B + Y + W = B + Y + R + B + Y$$
$$= R + C + C = R,$$

which uses the same rules, but takes an extra step as both G and W are replaced by primary colors.

All of these different combinations have a sum of R, so they are equal to each other, and interchangeable in gameplay:

$$Y + O = B + P = G + W = R.$$

In fact, every color in the game can be represented in many different ways, as the sum of two other colors; and any combinations adding up to the same color are interchangeable.

Every color can also be represented in many different ways, as the sum of three other colors; for example

$$Y + P + G = Y + R + B + B + Y$$
$$= R + C + C = R$$

and

$$O + P + W = R + Y + R + B + R + B + Y$$
$$= R + C + C + C = R$$

are interchangeable with all of the above combinations having sum R.

An easy technique for working out the sum of a set of colors is this:

1. Cancel the doubles from the set;

2. Replace each secondary color, or white, with a sum of the appropriate primary colors;

3. Cancel the doubles from this larger set of colors;

4. Replace the remaining colors with a single piece, if possible, or repeat these steps until only one piece remains (possibly a clear piece). The color of this piece is the sum of the original set.

As you become familiar with these rules and concepts, it is often possible to skip multiple steps in your mind, and you will begin to see many possibilities for different combinations at once.

Before playing, you should be familiar with these important combinations, and prove for yourself that they are true by the rules of the game:

$$R + O = Y, Y + O = R,$$
$$B + P = R, R + P = B,$$
$$B + G = Y, Y + G = B.$$

These show that a secondary color plus one of the primary colors composing it equals the other primary color composing it.

You should know, and prove for yourself, that

$$G + O = P, P + O = G, G + P = O,$$

i.e. that the sum of two secondary colors is equal to the third secondary color.

You should know, and prove for yourself, that adding the left side to the right side of any equation equals clear; for example

R + B + P = C.

You should experiment with sums involving white—it is the most versatile color in gameplay, as it contains all of the other colors.

Play around with the colors. See what happens if you add two or three colors together; see what combinations are equal to C; take a handful of pieces at random and find its sum. Soon you will discover your own combinations, and develop your own tricks.

AL-JABAR – Rules of Play

1. *Al-Jabar* is played by 2 to 4 people. The object of the game is to finish with the fewest game pieces in one's hand, as detailed below.

2. One player is the dealer. The dealer draws from a bag of 70 game pieces (10 each of the colors white, red, yellow, blue, orange, green, and purple), and places the 30 clear pieces in a location accessible to all players.

> *Note: Later in the game, it may happen that the clear pieces run out by rule 6. In this event, players may remove clear pieces from the center and place them in the general supply, taking care to leave a few in the center. If there are still an insufficient number, substitutes may be used, as the number of clears provided is not intended to be a limit.*

3. Each player is dealt 1 white game piece and 13 additional pieces, drawn at random, which remain visible to all throughout the game.

4. To initiate gameplay, one colored game piece, drawn at random from the bag, and one clear piece are placed upon the central game surface (called the "Center") by the dealer.

5. Beginning with the player to the left of dealer and proceeding counter-clockwise, each player takes a turn by exchanging any combination of 1, 2 or 3 pieces from his or her hand for a set of 1, 2 or 3 pieces from the Center having an equal sum of colors; or for a single clear piece, when appropriate.

The exception to this rule is the combination of 4 pieces R + B + Y + W (called a "spectrum"), which may be exchanged for a clear piece.

> *Note: Thus the shortest that a game may last is 5 moves, for a player may only reduce their hand by 3 marbles in a turn.*

The other players must then inspect the move and agree upon its validity, or point out any error. If in error, the current player must make a valid move before his or her turn is complete.

If a player having more than 3 game pieces in hand cannot make a valid move in a given turn, then he or she must draw additional pieces at random from the bag into his or her hand until a move can be made.

6. If a player's turn results in one or more pairs of like colors (such a pair is called a "double") occurring in the Center, then each such double is removed from the Center and discarded (or "cancelled"), to be replaced by a clear piece.

In addition, every other player must draw the same number of clear pieces as are produced by cancellations in this turn.

> *Note: Often near the end of the game, players will exchange three clear pieces from their hand for one from the center. Experienced players will often simply place two clears from their hand into the center, taking none, to simplify this move. It should be mentioned that this action is illegal if the player has only two clears in their hand, for then they still need to take one from the center.*

There are two exceptions to this rule: pairs of clear pieces are not cancelled from the Center; and, if a player's turn includes a double in the set of pieces placed from his or her hand, then the other players are not required to take additional clears.

> *Note: The goals of a player, during his or her turn, are to exchange the largest possible number of pieces from his or her*

hand, for the smallest number of pieces from the Center; and to create as many cancellations in the Center as possible, so as to require the other players to draw clear pieces.

7. A player may draw additional pieces as desired from the bag at random during his or her turn.

Note: If a player finds that his or her hand is composed mostly of a few colors, or requires a certain color for a particular move, this is often a wise idea.

8. A round of gameplay is complete when every player, starting with the first player, has taken a turn.

Two events signal that the game is in its final round.
(i) One player announces, immediately after his or her turn, that he or she has reduced his or her hand to one piece;
(ii) One player, having 3 or fewer pieces in hand, is unable to make a move resulting in a decrease in his or her total number of pieces.

In either case, the players who have not yet taken a turn in the current round are allowed to make their final moves.

When this final round is complete, the player with the fewest remaining pieces in hand is the winner. Tied players share the victory.