MA 213 Set Notation Review

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Introduction

This purpose of this review is to give an explicit explanation of notation that looks like $\{(x, y) \mid y \leq x^2, x > 0\}$ to the extent that it is used in MA 213. This notation is used to describe **sets**. A **set** is a mathematical "collection of objects" (the objects that are contained in a set are called **elements** of the set). In general, we can have sets with any type of object! But in MA 213 the sets we encounter are collections of real numbers, pairs or triplets of real numbers, or vectors.

Basic Notation

Curly brackets, $\{\}$ are used to denote sets. We can write a set by simply listing every object contained in the set. Here are some examples:

1. $\{1, 2, 3, 4, 5\}$ is a set containing five natural numbers.
2. {red,blue,purple} is a set containing three colors.
3. $\{0, -3.2, 2, 3, \pi, 10^{23}\}$ is a set containing six real numbers.
4. $\{(0, 0), (0, -1), (e, 1), (1, e)\}$ is a set containing four "pairs of real numbers" (which we can think of as four points in the xy plane).
5. $\{(0, 0, 0), (1, 2, 3)\}$ is a set containing two "triplets of real numbers" (which we can think of as two points in xyz space).
6. $\{3, (0, \pi), 2, (2, 2, 2), (0, 0, 0), (0, 0)\}$ is a set containing six objects, some which are numbers, some of which are points in the plane and some of which are points in xyz space.

To describe larger sets such as in interval in the real line, which has an infinite number of points, we can't list every point, so we have more notation. A vertical line, |, (or sometimes a colon, :) inside our curly brackets should be read as "such that". We can then write a set as,

$$\{\text{"type of object"} \mid \text{one or more conditions to be met by the object}\}$$

meaning we include in the set all objects of a type *such that* they meet the conditions we specify. Here are some examples:

1. $\{x \mid 0 \leq x < 1\}$, which we would read as "the set of all x (in our context, we'll assume we mean all real numbers, x), such that $0 \leq x$ and $x < 1$". We see then that this set is that same as the half open interval, $[0, 1)$.
2. $\{y \mid y^2 > 4, y \leq 10\}$, which we would read as "the set of all (real numbers) y, such that $y^2 > 4$ and $y < 10$". Check that this set is the same as $(-\infty, -2) \bigcup (2, 10]$.

Special Notation for some Special Sets

Some sets we refer to frequently and so we have symbols for them to make it easier. In MA 213, we use the following:

1. $\mathbb{R} = \{x \mid x$ is any real number$\}$ = the set of all real numbers.
2. $\mathbb{R}^2 = \{(x, y) \mid x$ and y are any real numbers$\}$ = all of the "xy plane".
3. $\mathbb{R}^3 = \{(x, y, z) \mid x$, y and z are any real numbers$\}$ = all of "xyz space".

Regions and Sets in $\mathbb{R}^2$ and $\mathbb{R}^3$

In chapter 15, set notation is used to describe regions in $\mathbb{R}^2$ and $\mathbb{R}^3$ over which we would like to integrate a function. Here are some examples of regions, curves and surfaces written in set notation. Try to sketch by hand and/or with a computer what each region is to try to get used to the notation:

1. $\{(x, y) \mid x \geq 0, y \leq 0\}$
2. $\{(x, y) \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}$
3. $\{(x, y) \mid 0 \leq x \leq 2, 0 \leq y \leq x\}$
4. $\{(x, y) \mid 0 \leq x \leq y, 0 \leq y \leq 3\}$
5. $\{(x, y) \mid x^2 + y^2 < 4\}$
6. $\{(x, y) \mid x^2 + y^2 = 4\}$
7. $\{(x, y) \mid x^2 + y^2 \leq 4\}$
8. $\{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3\}$
9. $\{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq 2, z = x + y\}$
10. $\{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq 2, -1 \leq z \leq x + y\}$
11. $\{(x, y, z) \mid x + y + z \leq 1, x \geq 0, y \geq 0, z \geq 0\}$