Differentials.

Calc I: $y = f(x)$

The differential

$$dy = f'(x)\,dx.$$

Think. $\Delta y \approx f'(x)\,\Delta x.$

Now: $z = f(x,y)$,

The differential.

$$dz = f_x(x,y)\,dx + f_y(x,y)\,dy$$

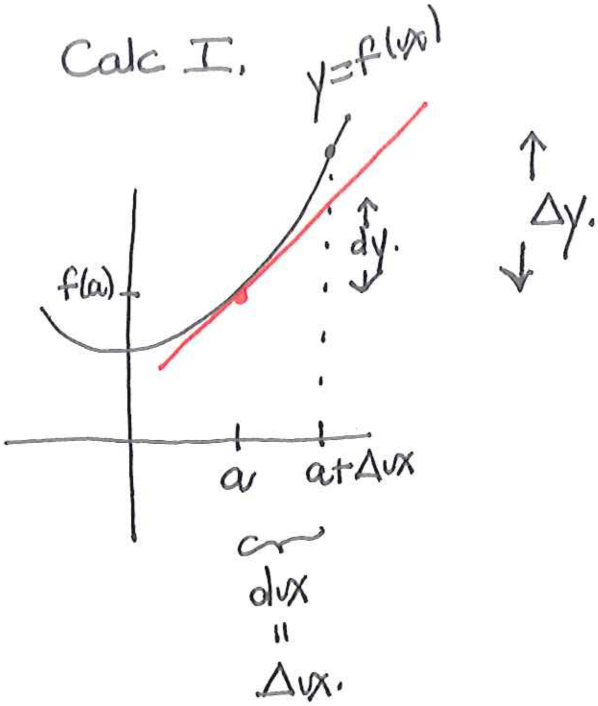
$$= \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy.$$

ex. volume of cone (later)

$$V = \frac{\pi r^2 h}{3} \quad \ldots$$

Picture

Calc I.



Tangent line

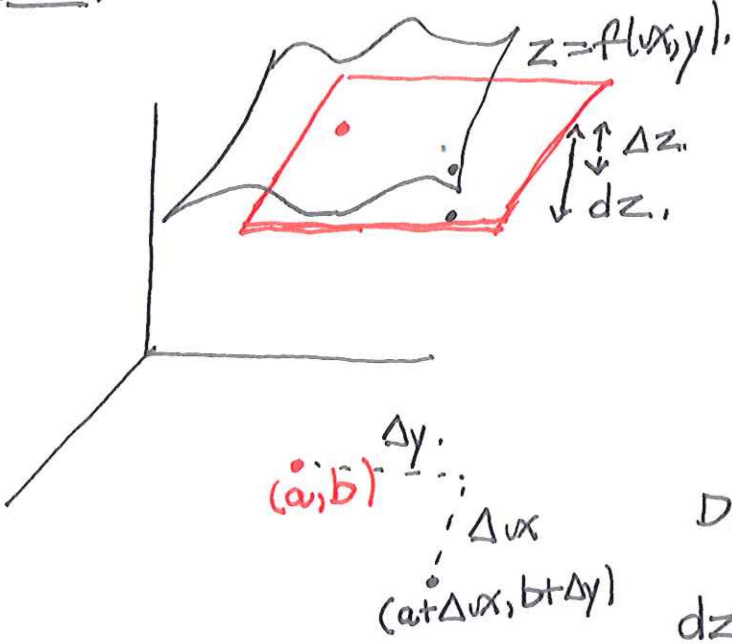
$$y - f(a) = f'(a)\,(x-a).$$

Diff'l.

$$dy = f'(x)\,dx.$$

Bring in stuffed animal & cardboard.

Now.



Tangent plane

$$z - f(a,b) = f_x(a,b)\,(x-a) + f_y(a,b)\,(y-b).$$

Differential.

$$dz = f_x(a,b)\,dx + f_y(a,b)\,dy.$$

ex. If

$$z = f(x,y) = x^2 + 4xy + y^3$$

find dz, the increment & diff'l,
Compare Δz & dz ~~for~~ (2,1) to (2.01, 1.04).

Sol'n

$$dz = f_x\,dx + f_y\,dy$$

$$= (2x+4y)\,dx + (4x+3y^2)\,dy.$$

$$\Delta z = f(2.01, 1.04) - f(2,1)$$

$$= (2.01)^2 + 4(2.01)(1.04) + (1.04)^3 - (2^2 + 4(2)\cdot(1) + 1^3)$$

$$= \ldots = .0526564$$

Now

$$dz = (2\cdot 2 + 4\cdot 1)(2.01-2) + (4\cdot 2 + 3\cdot 1^2)\cdot(1.04-1)$$

$$= 9\,(.01) + 11\,(.04)$$

$$= .09 + .44$$

$$\begin{array}{r} .09 \\ .44 \\ \hline .53 \end{array}$$

$$= .53.$$

ex. Let

$$z = f(x,y) = y \cdot \ln x.$$

Find dz.
Compare the increment Δz & the diff'l dz for (1,2) to (1.01, 2.4).

Sol'n

$$dz \; (= df) = f_x \, dx + f_y \, dy$$

$$= \frac{y}{x} \, dx + (\ln x) \, dy.$$

$$\Delta z = f(1.01, 2.4) - f(1,2)$$

$$= 2.4 \ln (1.01) - 2 \underbrace{\ln (1)}_{0}$$

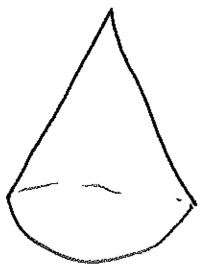
$$= .02388 \ldots$$

$$dz = \frac{2}{1} (1.01 - 1) + \ln (1) (2.4 - 2)$$

$$= 2 (.01) \quad + 0 \cdot (.4)$$

$$= .02.$$

ex.



right circular cone.

$$V = \frac{\pi r^2 h}{3}.$$

$r = 10$ cm

$h = 20$ cm.

error in measurement ± 0.1 cm.

Use differentials to estimate max error.

Sol'n.

$$dV = \frac{\partial V}{\partial r}\,dr + \frac{\partial V}{\partial h}\,dh.$$

$$= \frac{2\pi r h}{3}\,dr + \frac{\pi r^2}{3}\,dh.$$

$|\Delta r| \le .1$ Take $dr = .1$

$|\Delta h| \le .1$ $dh = .1$.

Then

$$dV = \frac{2\pi(10\text{cm})(20\text{cm})(.1\text{cm})}{3} + \frac{\pi}{3}(10\text{cm})^2(.1\text{cm})$$

$$= \frac{10\pi}{3}(40(.1)) + \frac{10\pi}{3}(10 \cdot .1)$$

$$\frac{140\pi}{3}\text{cm}^3 = \frac{10\pi}{3}(4+10)$$

Functions of More variables

$w = f(x, y, z)$.

Increment

$$\Delta w = f(x+\Delta x, y+\Delta y, z+\Delta z) - f(x, y, z).$$

Differential

$$dw = w_x\,dx + w_y\,dy + w_z\,dz$$

ex. Find diff'l of.

$$w = e^{x^2+y^2} \cdot z$$

$$dw = (2xe^{x^2+y^2} \cdot z)\,dx + (2ye^{x^2+y^2} \cdot z)\,dy + (e^{x^2+y^2})\,dz.$$

ex. ~~What error in volume of cube to store chemicals for is ±.1 cm.~~
How much measurement error in ~~measuring~~ producing a rectangular ~~cube~~-shaped box of dimensions $x \times y \times z$ if we want at most 1% error? where
$x = 1$ cm
$y = 2$ cm
$z = 3$ cm.

$$V = xyz.$$

$$dV = yz\,dx + xz\,dy + xy\,dz,$$

$$\Rightarrow dV = 2\,dx + 3\,dy + 6\,dz.$$

Say $dx = dy = dz$ $\quad \sim 11\,dx.$

Want $11\,dx < 1\%(6) = .06$

$$dx < \frac{.06}{11} = .00545$$