Math 213 - Exam III Review

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Homework

- Review for your exam, Wednesday, November 14
- Finish Homework C4

Unit III: Multiple Integrals

Lecture 24	Double Integrals over Rectangles
Lecture 25	Double Integrals over General Regions
Lecture 26	Double Integrals in Polar Coodinates
Lecture 27	Applications of Double Integrals
Lecture 28	Surface Area
Lecture 29	Triple Integrals
Lecture 30	Triple Integrals in Cylindrical Coordinates
Lecture 31	Triple Integrals in Spherical Coordinates
Lecture 32	Change of Variable in Multiple Integrals, Part I
Lecture 33	Change of Variable in Multiple Integrals, Part II
Lecture 34	Exam III Review

Goals of the Day

- Learn How to Ace Exam III!


Welcome!
Everything is fine.

Big Picture

In this unit we learned various techniques for reducing double and triple integrals to iterated integrals that can be computed "from the inside out"

Double Integrals

- Integrals over regions of type I and type II (xy coordinates)
- Integrals over regions defined by polar coordinates

Triple Integrals

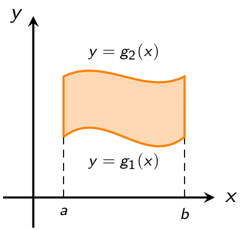
- Integrals over regions of type I (over xy plane), type II (over yz plane), type III (over xz plane)
- Integrals in cylindrical coordinates
- Integrals in spherical coordinates

Integrals in polar, cylindrical, and spherical coordinates are special cases of change of variables in multiple integrals. If R is a region in the xy plane, S is a region in the uv plane, and $T : (u, v) \to (x(u, v), y(u, v))$ a transformation that maps S to R, then

$$\int_R f(x, y)\, dA = \int_S f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du\, dv$$

How to Evaluate Iterated Integrals

What's on My Cheat Sheet? Double Integrals

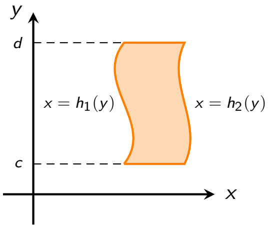


Type I:

$$\iint_R f(x,y)\,dA = \int_a^b \left(\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy \right)\,dx$$

where

$$R = \{(x,y) : a \leq x \leq b,\ g_1(x) \leq y \leq g_2(x)\}$$



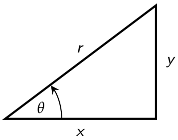
Type II:

$$\iint_R f(x,y)\,dA = \int_c^d \left(\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx \right)\,dy$$

where

$$R = \{(x,y) : c \leq y \leq d,\ h_1(y) \leq x \leq h_2(y)\}$$

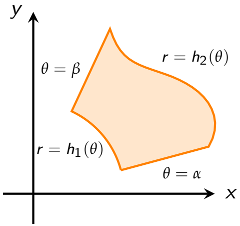
What's on My Cheat Sheet? Polar Integrals



Rectangular to Polar and Back Again

$$r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x}$$

$$x = r\cos\theta \qquad y = r\sin\theta$$



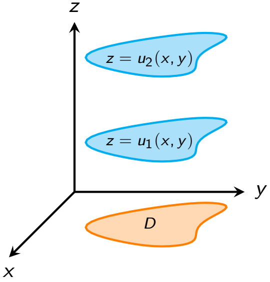
If

$$D = \{(r,\theta) : \alpha \le \theta \le \beta,\ \ h_1(\theta) \le r \le h_2(\theta)\}$$

then

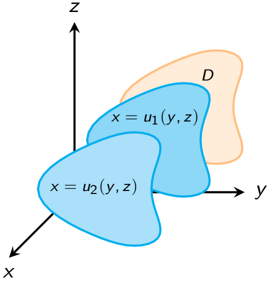
$$\iint_D f(x,y)\,dA = \int_\alpha^\beta \left(\int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta)\, r\, dr \right) d\theta$$

What's on My Cheat Sheet? Triple Integrals



Type I: D in xy plane, E bounded by $z = u_1(x, y)$ and $z = u_2(x, y)$

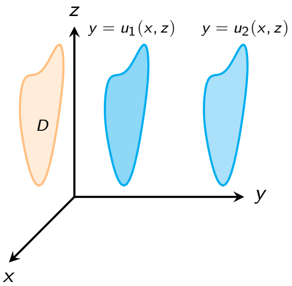
$$\iiint_E f(x, y, z)\, dV = \iint_D \left(\int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\, dz \right) dA$$



Type II: D in yz plane, E bounded by $x = u_1(y, z)$, $x = u_2(y, z)$

$$\iiint_E f(x, y, z)\, dV = \iint_D \left(\int_{u_1(y,z)}^{u_2(y,z)} f(x, y, z)\, dx \right) dA$$

What's on My Cheat Sheet? Triple Integrals



Type III: D in xz plane, E bounded by $y = u_1(x,z)$, $y = u_2(x,z)$

$$\iiint_E f(x,y,z)\,dV = \iint_D \left(\int_{u_1(x,z)}^{u_2(x,z)} f(x,y,z)\,dy \right) dA$$

What's On My Cheat Sheet? Cylindrical Coordinates

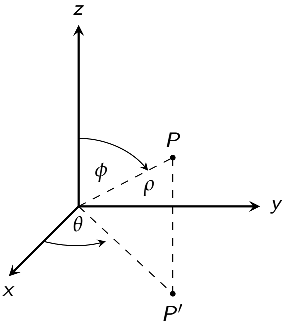
If

$$E = \{(r, \theta, z) : \alpha \leq \theta \leq \beta,\ h_1(\theta) \leq r \leq h_2(\theta),\ u_1(x, y) \leq z \leq u_2(x, y)\}$$

then we can evaluate

$$\iiint_E f(x, y, z)\, dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta$$

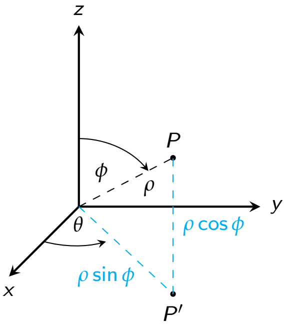
What's on My Cheat Sheet? Spherical Coordinates



Going over:

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\tan\theta = \frac{y}{x}$$
$$\cos\phi = \frac{z}{\rho}$$

What's on My Cheat Sheet? Spherical Coordinates



Going over:

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\tan\theta = \frac{y}{x}$$
$$\cos\phi = \frac{z}{\rho}$$

Coming back:

$$x = \rho\sin\phi\cos\theta$$
$$y = \rho\sin\phi\sin\theta$$
$$z = \rho\cos\phi$$

Triple Integrals, Spherical Coordinates

$$\iint_E f(x, y, z)\, dV =$$

$$\int_c^d \int_\alpha^\beta \int_a^b f(\rho \sin\phi \cos\theta, \rho \sin\phi \sin\theta, \rho cos\phi)\, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

if E is a spherical wedge

$$E = \{(\rho, \theta, \phi) : a \leq \rho \leq b,\ \alpha \leq \theta \leq \beta,\ c \leq \phi \leq d\}$$

What's on My Cheat Sheet? Change of Variables

Change of Variables in a Double Integral If T is a one-to-one transformation with nonzero Jacobian and $T : S \to R$, then

$$\iint_R f(x, y)\, dA = \iint_S f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du\, dv$$

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix}$$