1) $\vec{r}(t) = \langle t, t^2, t^3 \rangle$

$\vec{r}\,'(t) = \langle 1, 2t, 3t^2 \rangle$

2) $\vec{r}(t) = \langle \cos t, \sin t, 1 \rangle$

$\vec{r}\,'(t) = \langle -\sin t, \cos t, 0 \rangle$

3) $\vec{r}(t) = \langle 1, 2t, 3t^2 \rangle$

$$\int_0^1 \vec{r}(t)\, dt = \left\langle \int_0^1 1\, dt, \int_0^1 2t\, dt, \int_0^1 3t^2 dt \right\rangle$$

$$= \left\langle t\Big|_0^1, t^2\Big|_0^1, t^3\Big|_0^1 \right\rangle$$

$$= \langle 1, 1, 1 \rangle$$

$\vec{r}(t) = \langle t-2, t^2+1 \rangle$

$\vec{r}(-1) = \langle -3, 2 \rangle$

$\vec{r}\,'(t) = \langle 1, 2t \rangle$

$\vec{r}\,'(-1) = \langle 1, -2 \rangle$

$\vec{r}\,'(t)$ is the <u>tangent vector</u>

To the curve traced out by $\vec{r}(t)$

at $\vec{r}(t)$.

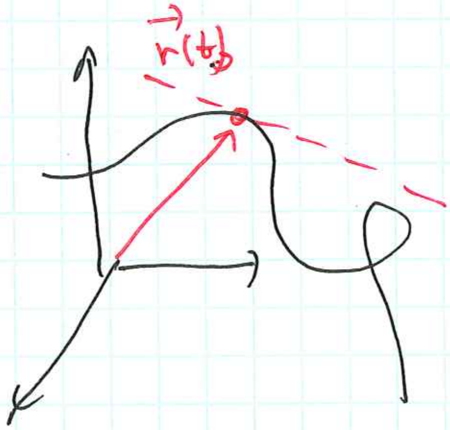
$\vec{u}(t) = \langle t^2, t^3 \rangle$

$v(t) = \langle \cos(t), \sin(t) \rangle$

$\vec{u} \cdot \vec{v} = t^2 \cos(t) + t^3 \sin(t)$

$$\frac{d}{dt}\, \vec{u}(t) \cdot \vec{v}(t) = \vec{u}\,'(t) \cdot v(t) + \vec{u}(t) \cdot \vec{v}\,'(t)$$

$$= \langle 2t, 3t^2 \rangle \cdot \langle \cos(t), \sin(t) \rangle + \langle t^2, t^3 \rangle \cdot \langle -\sin(t), \cos(t) \rangle$$



Point on the line : $\vec{r}(t_0)$

Vector along the line : $\vec{r}\,'(t_0)$

Tgt line to curve :

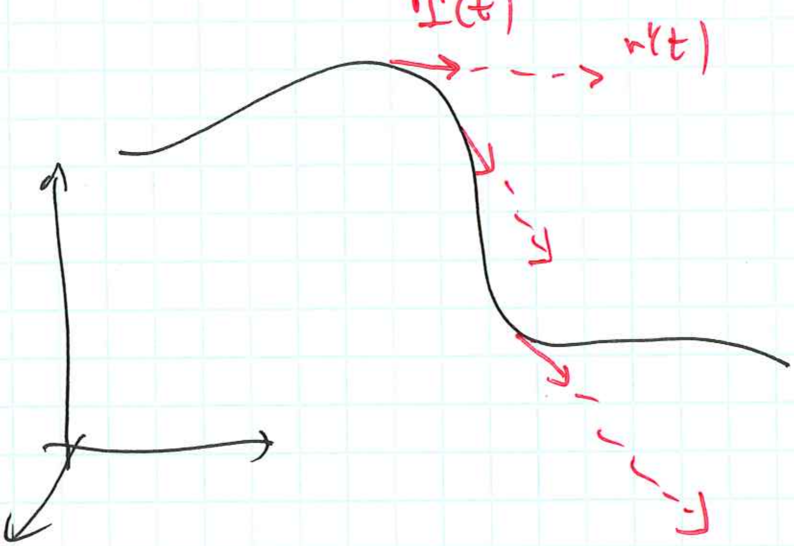
$t=0 \iff (0,1,0)$ ← Pt on line

$$\vec{r}\,'(t) = \langle 1, -e^{-t}, 2-2t \rangle$$

$\vec{r}\,'(0) = \langle 1, -1, 2 \rangle$ ← Vector along line

Parametric Eq'ns of line

$$x = 0 + 1t$$
$$y = 1 + (-1)t$$
$$z = 0 + 2t$$



1) $\vec{r}(t) = \langle t, t^2, t^3 \rangle$

$\vec{r}\,'(t) = \langle 1, 2t, 3t^2 \rangle$

$\vec{r}\,'(1) = \langle 1, 2, 3 \rangle$

$|\vec{r}\,'(1)| = \sqrt{1^2 + 2^2 + 3^2}$

$= \sqrt{14}$

$\vec{T}(1) = \dfrac{\vec{r}\,'(1)}{|\vec{r}\,'(1)|}$

$= \dfrac{1}{\sqrt{14}} \langle 1, 2, 3 \rangle$

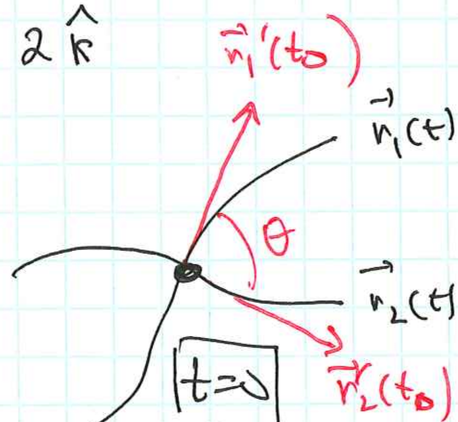
$\vec{r}\,''(t) = \langle 0, 2, 6t \rangle$

$$\vec{r}\,'(t) = \langle 1, 2t, 3t^2 \rangle$$

$$\vec{r}\,''(t) = \langle 0, 2, 6t \rangle$$

$$\vec{r}\,'(t) \times \vec{r}\,''(t) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix}$$

$$= (12t^2 - 6t^2)\,\hat{\imath} - 6t\,\hat{\jmath} + 2\hat{k}$$



$$\vec{r}_1(t) = \langle t, t^2, t^3 \rangle$$

$$\vec{r}_2(t) = \langle \sin t, \sin 2t, t \rangle$$

$$\vec{r}_1\,'(t) = \langle 1, 2t, 3t^2 \rangle \qquad \vec{r}_1\,'(0) = \langle 1, 0, 0 \rangle$$

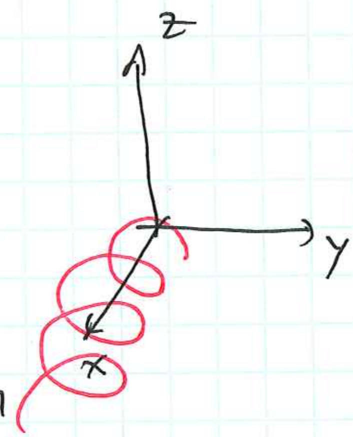
$$\vec{r}_2\,'(t) = \langle \cos t, 2\cos 2t, 1 \rangle \qquad \vec{r}_2\,'(0) = \langle 1, 2, 1 \rangle$$

$$|\vec{r}_1\,'(0)| = 1 \qquad |\vec{r}_2\,'(0)| = \sqrt{6} \qquad \vec{r}_1\,'(0) \cdot \vec{r}_2\,'(0) = 1$$

$$\cos\theta = \frac{1}{1\cdot\sqrt{6}}$$

$$\theta = \arccos\left(\frac{1}{\sqrt{6}}\right)$$

$$\vec{r}(t) = \langle t,\ 3\cos t,\ 3\sin t\rangle$$



$$\vec{r}\,'(t) = \langle 1,\ -3\sin t,\ 3\cos t\rangle$$

$$|\vec{r}\,'(t)| = \sqrt{1 + 9\sin^2 t + 9\cos^2 t}$$

$$= \sqrt{1+9}$$

$$= \sqrt{10}$$

$$s = \int_{-5}^{5} \sqrt{10}\,dt = \sqrt{10}\,(5-(-5)) = 10\sqrt{10}$$

$$s(t) = \int_0^t \sqrt{10}\,dt = \sqrt{10}\,t$$

Arc length function.