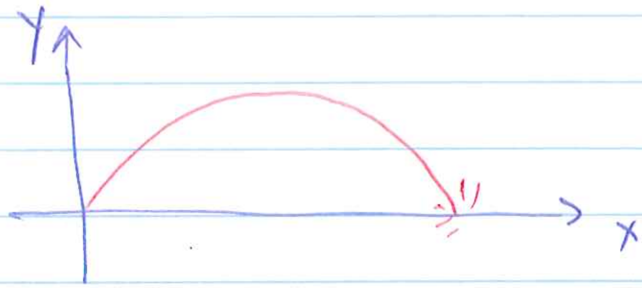
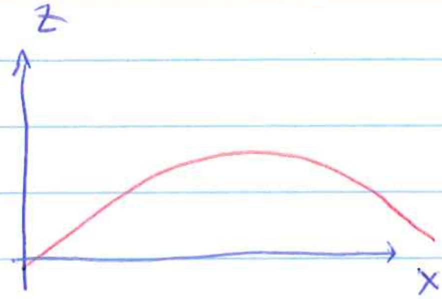
Projectile Motion:



1) $\hat{i}$ $\hat{j}$ method

$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

* 2)



$$\vec{r}(t) = x(t)\,\hat{i} + z(t)\,\hat{k}$$

#1 $\vec{r}\,'(t) = \langle 32,\ 32 - 32t \rangle$

$$= 32\hat{i} + (32 - 32t)\hat{j} = \vec{v}(t)$$

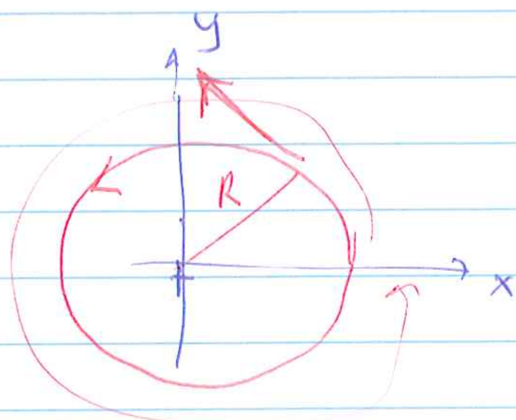
$\vec{r}\,''(t) = \langle 0, -32 \rangle$

$$= 0\hat{i} - 32\hat{j} = \vec{a}(t)$$

$$\vec{r}(t) = \langle R\cos(2\pi t/T),\ R\sin(2\pi t/T)\rangle$$

R = radius of the circle

T = period



$$\vec{v}(t) = \vec{r}\,'(t)$$

$$= \left\langle -R\sin\left(\frac{2\pi t}{T}\right)\cdot\frac{2\pi}{T},\ \frac{2\pi}{T}R\cos\left(\frac{2\pi t}{T}\right)\right\rangle$$

$$= \frac{2\pi R}{T}\left\langle -\sin\left(\frac{2\pi t}{T}\right),\ \cos\left(\frac{2\pi t}{T}\right)\right\rangle$$

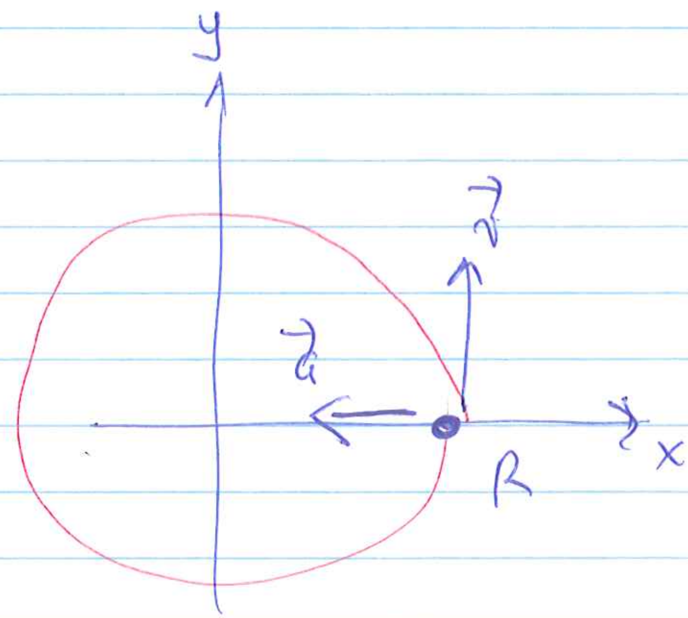
$$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$$

$$= \frac{2\pi R}{T}\cdot\frac{2\pi}{T}\left\langle -\cos\left(\frac{2\pi t}{T}\right),\ -\sin\left(\frac{2\pi t}{T}\right)\right\rangle$$

$$\vec{r}(t) = R \left\langle \cos\left(\frac{2\pi t}{T}\right), \sin\left(\frac{2\pi t}{T}\right) \right\rangle$$

$$\vec{v}(t) = \frac{2\pi R}{T} \left\langle -\sin\left(\frac{2\pi t}{T}\right), \cos\left(\frac{2\pi t}{T}\right) \right\rangle$$

$$\vec{a}(t) = \frac{4\pi^2 R}{T^2} \left\langle \cancel{\sin} -\cos\left(\frac{2\pi t}{T}\right), -\sin\left(\frac{2\pi t}{T}\right) \right\rangle$$

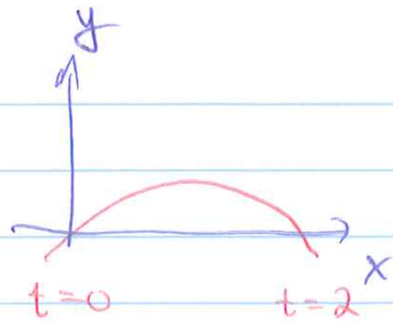


$t = 0$

$$\vec{r}(0) = R \langle 1, 0 \rangle$$

$$\vec{v}(0) = \frac{2\pi R}{T} \langle 0, 1 \rangle$$

$$\vec{a}(0) = -\frac{4\pi^2 R}{T^2} \langle -1, 0 \rangle$$



Projectile

$$a(t) = \vec{r}''(t) = \langle 0, -32 \rangle$$

$$v(t) = \vec{r}'(t) = \langle 32, 32 - 32t \rangle$$

$$\vec{r}(t) = \langle 32t, 32t - 16t^2 \rangle$$

distance : feet

time : sec.

Accel: $-32 \dfrac{ft}{sec^2}$

Time that projectile hits ground:

$$32t - 16t^2 = 0$$

$$16t(2 - t) = 0$$

$$t = 0, \quad t = 2$$

Speed when it hits:

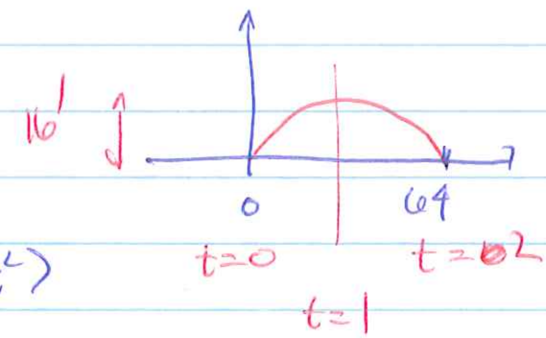
$$\vec{v}(2) = \langle 32, 32 - 32 \cdot 2 \rangle = \langle 32, -32 \rangle$$

$$\text{speed} = |\vec{v}(2)|$$

$$= |\langle 32, -32\rangle|$$

$$= \sqrt{32^2 + 32^2}$$

$$= 32\sqrt{2}$$



$$\vec{r}(t) = \langle 32t, 32t - 16t^2\rangle$$

$$x(t) = 32t$$

$$x(2) = 32 \cdot 2 = 64 \text{ ft}$$

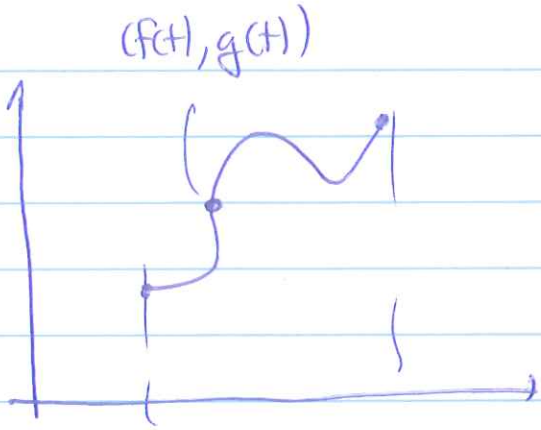
(Distance travelled)

Max height:

$$y(t) = 32t - 16t^2$$

$$y'(t) = 32 - 32t = 0 \quad \text{when } t = 1$$

$$y(1) = 32 - 16 = 16'$$



$x = f(t)$

$y = g(t)$

$a \le t \le b$

$$L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2}\, dt$$

The curve is a plane curve

$$\vec{r}(t) = f(t)\,\hat{i} + g(t)\,\hat{j}$$

velocity $\quad \vec{r}\,'(t) = f'(t)\,\hat{i} + g'(t)\,\hat{j}$

speed $\quad |\vec{r}\,'(t)| = \sqrt{f'(t)^2 + g'(t)^2}$

Distance travelled from $t=a$ to $t=b$

$$\int_a^b |\vec{r}\,'(t)|\, dt = \int_a^b \sqrt{f'(t)^2 + g'(t)^2}\, dt$$

$$x(t) = \cos(t) \qquad x'(t) = -\sin(t)$$

$$y(t) = \sin(t) \qquad y'(t) = \cos(t)$$

$$z(t) = \ln(\cos(t)) \qquad z'(t) = \frac{1}{\cos(t)}(-\sin t)$$

$$= -\tan t$$

$$\int_0^{\pi/4} \sqrt{\sin^2(t) + \cos^2 t + \tan^2 t}\; dt$$

$$= \int_0^{\pi/4} \sqrt{1 + \tan^2 t}\; dt$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 t}\; dt$$

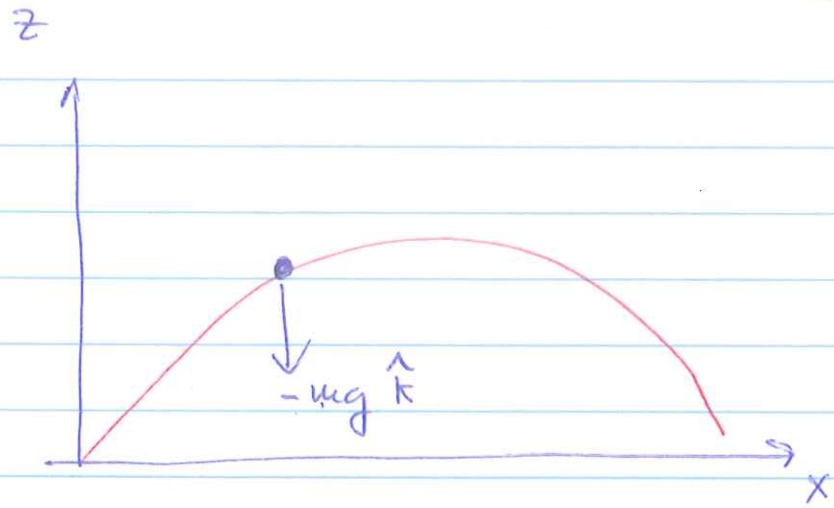
$$= \int_0^{\pi/4} \sec t \; dt$$

$$= \overset{\ln}{\cancel{\log}}(\tan(t) + \sec(t)) \Big|_0^{\pi/4}$$

$$= \overset{\ln}{\cancel{\log}}\left(1 + \frac{2}{\sqrt{2}}\right) - \ln(\cancel{0} + 1) \qquad 0$$

$$t = \pi/4 \qquad = \log\left(1 + \frac{2}{\sqrt{2}}\right)$$

$\frac{d}{dt}(m\vec{v})$ 8

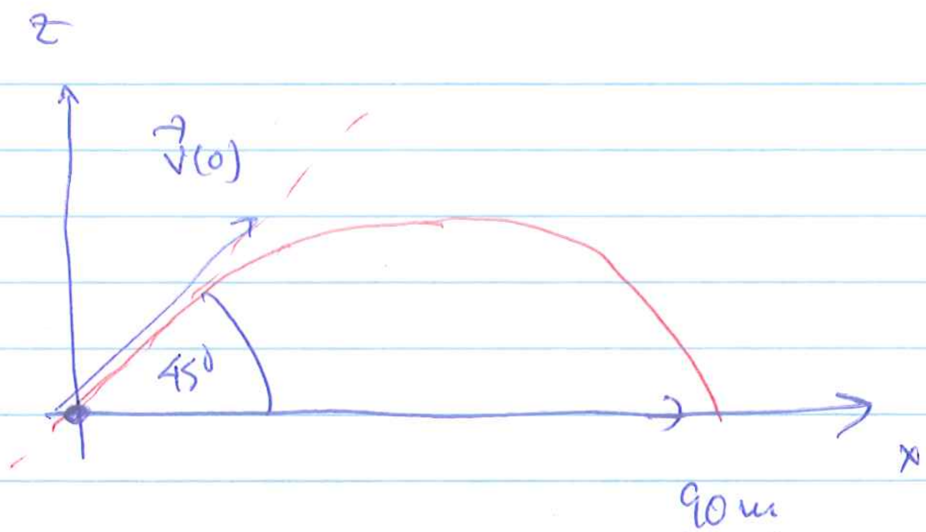


$$\vec{F} = -\cancel{m}g\hat{k} = \cancel{m}\vec{a}$$

$$\vec{a} = -g\hat{k}$$

$g = 32\ ft/sec^2$ FPS

$g = 9.8\ m/sec^2$ Metric



$$\vec{r}(0) = 0\hat{\imath} + 0\hat{k}$$

$$\vec{r}''(t) = -9.8 \ m/sec^2 \ \hat{k}$$

$$\vec{v}(0) = \vec{r}'(0) = v_0 \left(\underbrace{\frac{\sqrt{2}}{2}}_{\cos(45^0)} \hat{\imath} + \underbrace{\frac{\sqrt{2}}{2}}_{\sin(45^0)} \hat{k} \right)$$

$$\vec{v}(t) = \vec{v}(0) + \int_0^t \vec{a}(s)\,ds$$

$$= v_0 \left(\frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{k} \right) + \int_0^t (-9.8)\hat{k}\,ds$$

$$= \frac{\sqrt{2}}{2} v_0 \hat{\imath} + \left(v_0 \frac{\sqrt{2}}{2} - 9.8\,t \right) \hat{k}$$

$$\vec{r}(t) = \frac{\sqrt{2}}{2} v_0 t \,\hat{i} + \left(v_0 \frac{\sqrt{2}}{2} t - 9.8 \frac{t^2}{2} \right) \hat{k}$$

Time hits the ground:

$$v_0 \frac{\sqrt{2}}{2} t - 9.8 \frac{t^2}{2} = 0$$

$$t \left(v_0 \frac{\sqrt{2}}{2} - \frac{9.8 t}{2} \right) = 0$$

$$v_0 \frac{\sqrt{2}}{2} - \frac{9.8 t}{2} = 0$$

$$\frac{9.8 t}{\cancel{2}} = v_0 \frac{\sqrt{2}}{\cancel{2}}$$

$$\boxed{t_h = \frac{\sqrt{2}\, v_0}{9.8}}$$

$$x(t_h) = 90 \text{ m}$$

$$\boxed{v_0 \left(\frac{\sqrt{2}\, v_0}{9.8} \right) = 90 \text{ m}} \quad \leftarrow \text{solve for } v_0$$