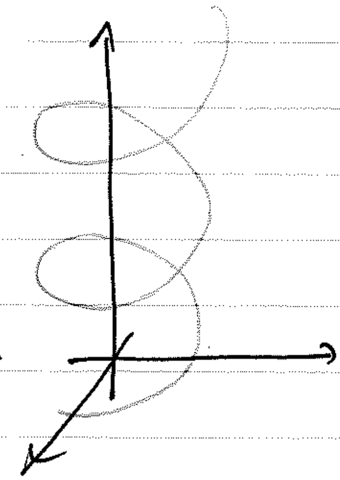
Parameterization by Arc Length

$$\vec{r}(t) = \langle \cos(2t), \sin(2t), t \rangle$$



$$\vec{r}\,'(t) = \langle -2\sin(2t), 2\cos(2t), 1 \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{4\sin^2(2t) + 4\cos^2(2t) + 1}$$

$$= \sqrt{5}$$

$$s(t) = \int_0^t |\vec{r}\,'(u)|\, du$$

$$= \int_0^t \sqrt{5}\, du$$

$$= \sqrt{5}\, t$$

$$\underline{s = \sqrt{5}\, t}$$

solve for t in terms of s:

$$\boxed{t = \cancel{5}\ \frac{s}{\sqrt{5}}}$$

Arc length (2)

$$\vec{r}(t) = \langle \cos(2t), \sin(2t), t \rangle$$

Substitute $t = \frac{s}{\sqrt{5}}$

$$\vec{r}(s) = \left\langle \cos\left(\frac{2s}{\sqrt{5}}\right), \sin\left(\frac{2s}{\sqrt{5}}\right), \frac{s}{\sqrt{5}} \right\rangle$$

(d) (7 points) Find a vector function that represents the intersection of the paraboloid $z = 4x^2 + y^2$ and the parabolic cylinder $y = x^2$. *Hint*: First parameterize x and y to move along the parabola $y = x^2$.

Let $x(t) = t$

Then $y(t) = x(t)^2 = t^2$ (parabolic cylinder)

and $z(t) = 4x(t)^2 + y(t)^2$ (paraboloid)

$= 4t^2 + (t^2)^2$

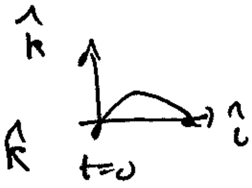
$= 4t^2 + t^4$

Hence

$\vec{r}(t) = \langle t, t^2, 4t^2 + t^4 \rangle$

6. (Motion in Space - 18 points)

A projectile is fired with initial speed of 200 m/s and angle of elevation 60°. Recall that the acceleration due to gravity is 9.8 m/sec^2.



(a) (3 points) Find the initial velocity vector $\mathbf{v}_0$.

$$\vec{v}_0 = 200\cos 60^\circ\,\hat{i} + 200\sin 60^\circ\,\hat{k} = 100\hat{i} + 100\sqrt{3}\,\hat{k}$$

(b) (3 points) Find the equation for $\mathbf{r}(t)$, the position of the projectile as a function of time.

$$\vec{r}''(t) = -9.8\,\hat{k}$$
$$\vec{r}'(t) = -9.8t\,\hat{k} + \vec{v}_0 = 100\hat{i} + (100\sqrt{3} - 9.8t)\hat{k}$$

$$\vec{r}(t) = \vec{r}(0) + 100t\,\hat{i} + (100\sqrt{3}\,t - 4.9t^2)\hat{k}$$

(c) (3 points) Find the time of impact.

$$100\sqrt{3}\,t - 4.9t^2 = 0$$
$$t(100\sqrt{3} - 4.9t) = 0$$
$$t = 100\sqrt{3}/4.9$$
$$t = 35.34$$

(d) (3 points) Find the maximum height reached.

Use $t = 35.34/2 \simeq 17.67$

$$z(17.67) = 100\sqrt{3}(17.67) - 4.9(17.67)^2 = 1530.61\ m$$

(e) (3 points) Find an expression for $\mathbf{r}'(t)$, the velocity of the projectile.

$$r'(t) = 100\hat{i} + (100\sqrt{3} - 9.8t)\hat{k}$$

(f) (3 points) Find the speed of the projectile at impact.

$$\text{Speed} = |\vec{r}'(t)|$$
$$r'(35.34) = 100\hat{i} + (100\sqrt{3} - 9.8\cdot 35.34)\hat{k} = 100\hat{i} + -173.13\,\hat{k}$$
$$|r'(35.34)| \simeq 200\ m/sec.$$