Math 213 - Exam I Review

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Homework

- Your test is tonight, February 6 at 5:00 PM in Room CB 106
- Remember that you're allowed an 8-1/2" $\times$ 11" sheet of paper with notes on *both sides*
- Be sure to bring your student ID and to arrive 10 minutes early so that we can start the exam on time and get you out by 7 PM
- Remember that Webwork A6 on 13.3-13.4 is due tonight!

Unit I: Geometry and Motion in Space

Lecture 1	Three-Dimensional Coordinate Systems
Lecture 2	Vectors
Lecture 3	The Dot Product
Lecture 4	The Cross Product
Lecture 5	Equations of Lines and Planes, Part I
Lecture 6	Equations of Lines and Planes, Part II
Lecture 7	Cylinders and Quadric Surfaces
Lecture 8	Vector Functions and Space Curves
Lecture 9	Derivatives and integrals of Vector Functions
Lecture 10	Motion in Space: Velocity and Acceleration
Lecture 11	Exam 1 Review

Topics of the Day

- What's on My Cheat Sheet?
- How to Do Multiple Choice Questions
- How to Write a Good Free Response
- Hot Topics, including:
  - Parameterizing by arc length
  - Projectile problems

Dot Product, Cross Product, Triple Product

$\mathbf{a} \cdot \mathbf{b}$	$a_1 b_1 + a_2 b_2 + a_3 c_3$	$\lvert\mathbf{a}\rvert \lvert\mathbf{b}\rvert \cos\theta$	Zero if **a**, **b** orthogonal
$\mathbf{a} \times \mathbf{b}$	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$	$\lvert\mathbf{a} \times \mathbf{b}\rvert = \lvert\mathbf{a}\rvert \lvert\mathbf{b}\rvert \sin\theta$	Zero if **a**, **b** parallel
$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$	$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$		Zero if **a**, **b**, **c** coplanar

$\lvert\mathbf{a} \times \mathbf{b}\rvert$ is area of a parallelogram spanned by **a**, **b**

$\lvert\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})\rvert$ is volume of parallelipiped spanned by **a**, **b**, **c**

Component of **b** in **a** direction: $\dfrac{\mathbf{b} \cdot \mathbf{a}}{\lvert\mathbf{a}\rvert}$

Vector projection of **b** in **a** direction: $\dfrac{\mathbf{b} \cdot \mathbf{a}}{\lvert\mathbf{a}\rvert^2}\mathbf{a}$

Lines

If (x_0, y_0, z_0) is a point on the line and $\mathbf{v} = \langle a, b, c \rangle$ points along the line:

Parametric equations: $x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$

Symmetric equations: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Two lines $\mathbf{r}_1(s) = (x_1, y_1, z_1) + s\mathbf{v}_1$ and $\mathbf{r}_2(t) = (x_2, y_2, z_2) + t\mathbf{v}_2$ are:

- parallel if $\mathbf{v}_1$ is parallel to $\mathbf{v}_2$
- intersecting if $\mathbf{r}_1(s) = \mathbf{r}_2(t)$ for some values of s and t
- skew if none of the above

Planes

If (x_0, y_0, z_0) is a point on the plane and $\mathbf{n} = \langle a, b, c \rangle$ is a vector normal to the plane:

$$ax + by + cz = d$$

where d is determined by substituting $(x, y, z) = (x_0, y_0, z_0)$ into the left-hand side

Two planes with normals $\mathbf{n}_1$ and $\mathbf{n}_2$ are:

- *parallel* if $\mathbf{n}_1$ and $\mathbf{n}_2$ are parallel
- *intersecting* if their normal vectors are not parallel. The vector $\mathbf{n}_1 \times \mathbf{n}_2$ points along the line of intersection

Cylinders A cylinder consists of a curve translated along a line parallel to one of the x-, y, or z axes (the "missing variable"). Examples: $y^2 + z^2 = 1$, $z = \sin(x)$

Quadric Surfaces

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Ellipsoid
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	Hyperboloid (One Sheet)
$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Hyperboloid (Two Sheets)
$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Cone
$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Elliptic paraboloid
$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$	Hyperbolic paraboloid

You can determine the graph of a quadric surface by finding its *traces* in planes $x = k$, $y = k$, $z = k$

Vector Functions

The function $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$, $a \leq t \leq b$ traces out a *space curve* C

The tangent line to $\mathbf{r}(t)$ at $t = t_0$ is the line containing the point $\mathbf{r}(t_0)$ in the direction of $\mathbf{r}'(t_0)$

The *velocity* of a space curve is $\mathbf{r}'(t)$, and the *speed* is $|\mathbf{r}'(t)|$

The *arc length* along C from $t = a$ to $t = b$ is $\int_a^b |\mathbf{r}'(t)|\, dt$

The *arc length function* for C is $s(t) = \int_a^t |\mathbf{r}'(u)|\, du$

Projectile Problems

For a projectile that starts at position $\mathbf{r}_0$, velocity $\mathbf{v}_0 = v_x\mathbf{i} + v_z\mathbf{k}$, we can find the motion by integrating

$$\mathbf{a}(t) = -g\mathbf{k}$$

to get

$$\mathbf{v}(t) = v_x\mathbf{i} + (v_z - gt)\mathbf{k}$$

and

$$\mathbf{r}(t) = \mathbf{r}_0 + (v_x t)\,\mathbf{i} + \left(v_z t - \frac{1}{2}gt^2\right)\,\mathbf{k}$$

Here g is the acceleration due to gravity:

$g = 32 \text{ ft}/\text{sec}^2$	FPS units
$g = 9.8 \text{ m}/\text{sec}^2$	MKS units

Multiple Choice Strategy

- Don't try to guess – compute the right answer!
- Make a record of your work so that you can go back and re-check your calculations
- Remember that just because your answer appears as a choice doesn't mean it's the right one!
- Remember to recheck your *work* and your *answer*!

I'll illustrate with some questions from the multiple choice practice exam.

Parameterizing by Arc Length

The *arc length function* for a curve C is

$$s(t) = \int_a^t |\mathbf{r}'(u)|\, du$$

You can parameterize a curve by arc length if you solve for t in terms of s and substitute for t in the formula for the curve.

Example: Parameterize the helix curve

$$\mathbf{r}(t) = \langle \cos(2t), \sin(2t), t \rangle, \quad 0 \leq t \leq 4\pi$$

by arc length.

Projectile Problems

A batter hits a baseball 3ft above the ground toward the center field fence, which is 10 ft high and 400 ft from home plate. The ball leaves the bat with speed 115 ft/sec at an angle of 50° above the horizontal. It is a home run? (that is, does the ball clear the fence?)

Open Mike

Good Luck on Exam I!