Math 213 - Functions of Two Variables

Peter A. Perry

University of Kentucky

February 8, 2019

Homework

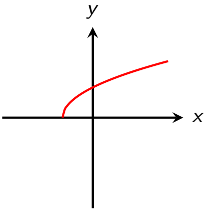
- Your exam scores should be in Canvas
- You will get your exam papers back in Tuesday's recitation
- If you have any grading concerns, please turn your papers back to your TA at the end of Tuesday's recitation. We can't accept regrading requests after this point.
- Be sure and keep up with the posted revised schedule for reading and homework
- Re-read section 14.1 and work on practice problems from Stewart: section 14.1, 9-19 (odd), 32, 36, 45, 49, 53

Unit II: Differential Calculus of Several Variables

Goals of the Day

- Know how to find the domain of a function of several variables
- Know how to graph a function of two variables in three-dimensional space
- Know how to find the level curves of a function of two variables and to match the graph of a function with its contour plot
- Know how to find level surfaces of a function of three variables

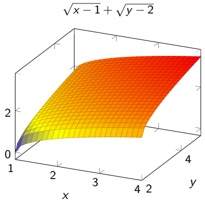
One Variable versus Two Variables



A function of one variable is a map $f : I \to \mathbb{R}$ where the domain, I, is a subset of the real line

Example: $f(x) = \sqrt{1+x}$, $I = (-1, \infty)$

The *graph* of f is the set of points $(x, f(x))$ in the xy plane, where $x \in I$



A function of two variables is a map $f : U \to \mathbb{R}$ where the domain U is a subset of $\mathbb{R}^2$.

Example: $f(x, y) = \sqrt{x-1} + \sqrt{y-2}$,

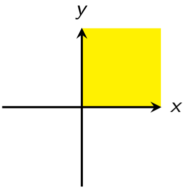
$$U = \{(x, y) : x \geq 1, y \geq 2\}$$

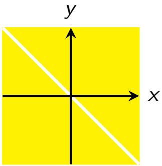
The *graph* of f is the set of points $(x, y, f(x, z))$ in the xyz plane

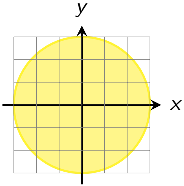
Match the following functions with the graphs of their domains in the xy-plane.

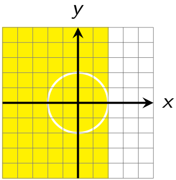
$$f(x,y) = \sqrt{9 - x^2 - y^2} \qquad f(x,y) = \frac{x-y}{x+y}$$

$$f(x,y) = \frac{\ln(2-x)}{4 - x^2 - y^2} \qquad f(x,y) = \sqrt{x} + \sqrt{y}$$







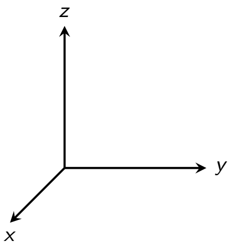


Linear Functions

A function of the form $f(x, y) = ax + bx + c$ for numbers a, b, and c is a *linear function*. Its graph is a plane:

$$z = ax + by + c \Rightarrow ax + by - z = c$$

You already know how to graph this!

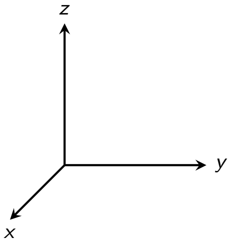


Linear Functions

A function of the form $f(x, y) = ax + bx + c$ for numbers a, b, and c is a *linear function*. Its graph is a plane:

$$z = ax + by + c \Rightarrow ax + by - z = c$$

You already know how to graph this!



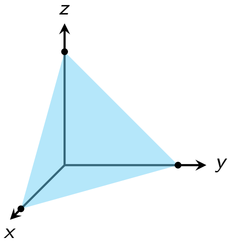
Find the graph of $f(x, y) = 2 - x - y$

Linear Functions

A function of the form $f(x, y) = ax + bx + c$ for numbers a, b, and c is a *linear function*. Its graph is a plane:

$$z = ax + by + c \Rightarrow ax + by - z = c$$

You already know how to graph this!



Find the graph of $f(x, y) = 2 - x - y$

$$x + y + z = 2$$

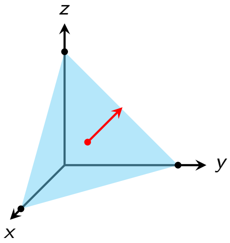
$(2, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$ all lie on this plane

Linear Functions

A function of the form $f(x, y) = ax + bx + c$ for numbers a, b, and c is a *linear function*. Its graph is a plane:

$$z = ax + by + c \Rightarrow ax + by - z = c$$

You already know how to graph this!



Find the graph of $f(x, y) = 2 - x - y$

$$x + y + z = 2$$

$(2, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$ all lie on this plane

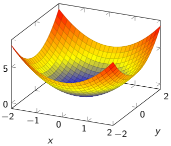
The normal vector is $\langle 1, 1, 1 \rangle$

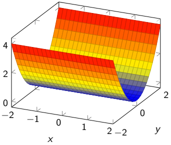
Quadratic Functions

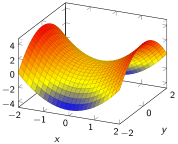
Everything you know about cylinders and quadric surfaces $z = f(x, y)$ tells you something about graphs. Can you match these functions to their graphs?

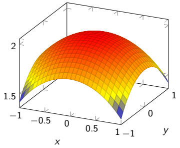
$$f(x,y) = y^2 \qquad\qquad f(x,y) = x^2 - y^2$$

$$f(x,y) = \sqrt{4 - x^2 - y^2} \qquad\qquad f(x,y) = x^2 + y^2$$







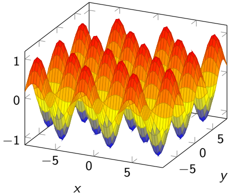


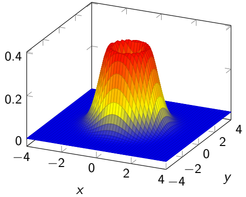
Common Sense and Connection

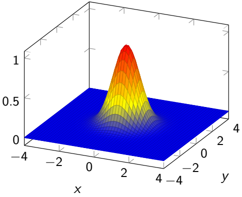
Can you match these functions with their graphs?

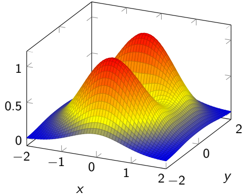
$$f(x,y) = \sin(x)\cos(y) \qquad f(x,y) = \exp(-x^2 - y^2)$$

$$f(x,y) = (x^2 + y^2)e^{-(x^2+y^2)} \qquad f(x,y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$$



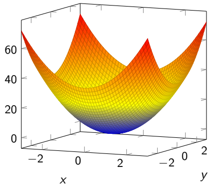






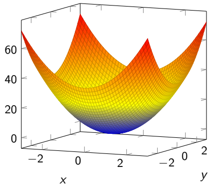
Level Curves

Definition The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant in the range of f.



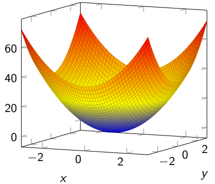
Level Curves

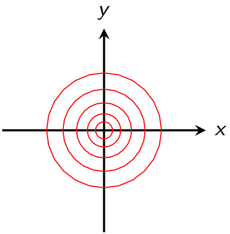
Definition The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant in the range of f.



Level Curves

Definition The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant in the range of f.



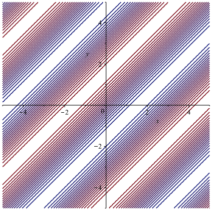


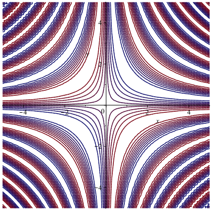
- What is the range of the function $f(x, y) = x^2 + y^2$?
- Describe the level curves of this function

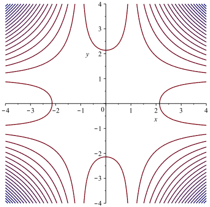
Contour Plots

A **contour plot** of a function shows a number of level curves. Can you match these functions with their graphs and contour plots?

$$f(x,y) = \sin(xy) \qquad f(x,y) = (1-x^2)(1-y^2) \qquad f(x,y) = \sin(x-y)$$







You Already Know About Contour Plots

Let's examine a topo map from the Great Smoky Mountains National Park courtesy of the United States Geological Survey (USGS)

Functions of Three Variables

A function of three variables is a map $f : V \to \mathbb{R}$ where the domain V is a subset of $\mathbb{R}^3$

Find the domain and range of these functions of three variables

1. $f(x, y, z) = x^2 + y^2 + z^2$
2. $f(x, y, z) = \sqrt{9 - x^2 - y^2 - z^2}$
3. $f(x, y, z) = x + y + z$

Definition The **level surfaces** of a function f of three variables are the surfaces with equation $f(x, y, z) = k$ where k is a constant in the range of f.

Determine the level surfaces of the the following functions:

1. $f(x, y, z) = x^2 + y^2 + z^2$
2. $f(x, y, z) = \sqrt{9 - x^2 - y^2 - z^2}$
3. $f(x, y, z) = x + y + z$