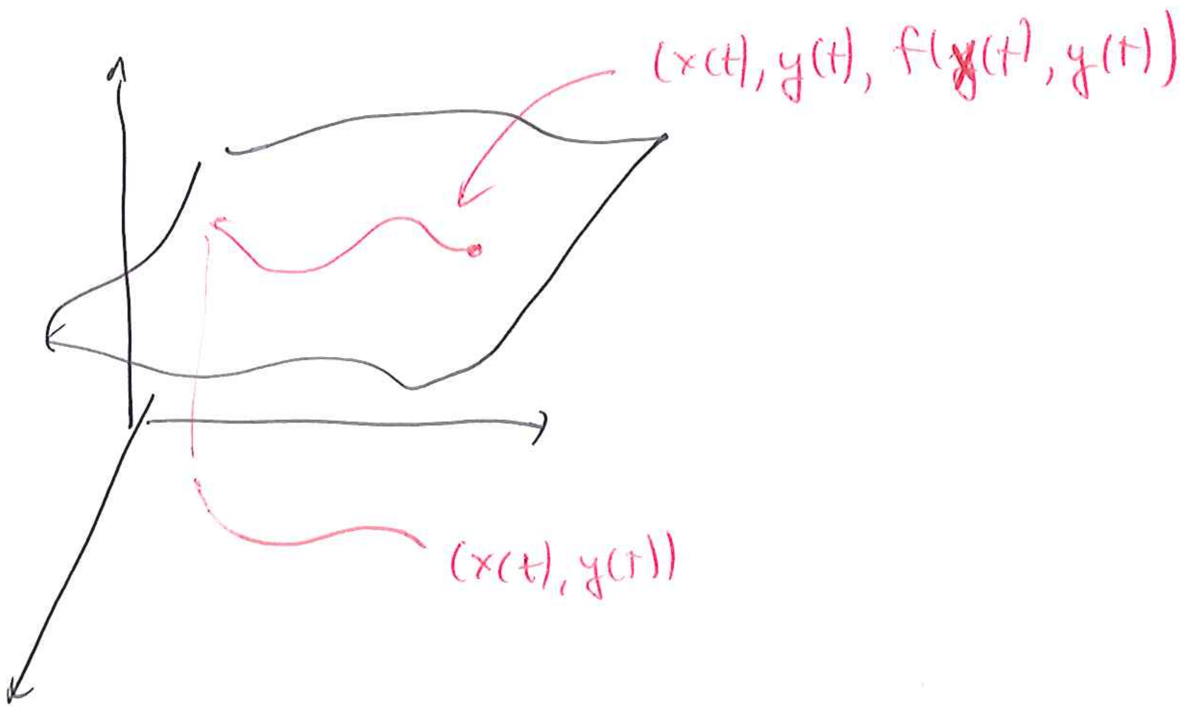
2/18/2019 (1)

$$z = f(x,y) \qquad x = g(t),\ y = h(t)$$

$$\frac{d}{dt} f(x(t), y(t)) = \frac{\partial f}{\partial x}(x(t), y(t)) \cdot \frac{dx}{dt}$$

$$+ \frac{\partial f}{\partial y}(x(t), y(t)) \cdot \frac{dy}{dt}$$



$f_x(1,3) = -2$ $\qquad$ $f_y(1,3) = 4$

$x(t) = t$ $\qquad$ $y(t) = 3t$

At $=1$, $\quad x(1) = 1$ $\qquad$ $y(1) = 3$

$\underline{x'(t) = 1}$ $\qquad$ $\underline{\underline{y'(t) = 3}}$

$\frac{d}{dt} f(x(t), y(t))$ at $t=3$:

$$\frac{\partial f}{\partial x}(1,3) \cdot \underline{1} + \frac{\partial f}{\partial y}(1,3) \cdot \underline{\underline{3}} =$$

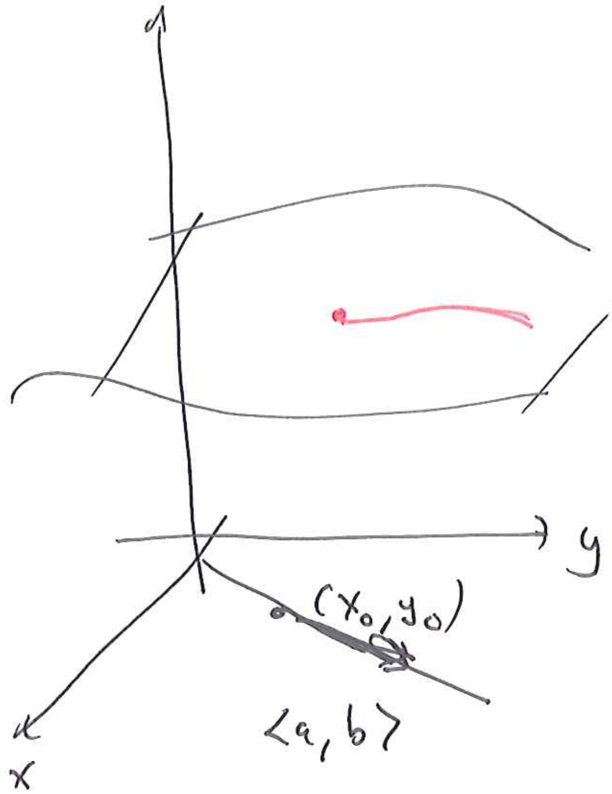
$$(-2) \cdot 1 + 4 \cdot 3 =$$

$$-2 + 12 = 10$$

$f(x, y)$

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$



$$\frac{d}{dt} f(x(t), y(t)) =$$

$$\frac{\partial f}{\partial x}(x(t), y(t)) \cdot a \; +$$

$$\frac{\partial f}{\partial y}(x(t), y(t)) \cdot b$$

$t = 0$:

$$\boxed{\begin{aligned} \frac{d}{dt} f(x(t), y(t)) \Big|_{t=0} &= \\ \frac{\partial f}{\partial x}(x_0, y_0) \cdot a \; &+ \\ \frac{\partial f}{\partial y}(x_0, y_0) \cdot b & \end{aligned}}$$

1. $f(x,y) = xy^3 - x^2$

$(x_0, y_0) = (1, 2)$

$\underset{\sim}{u} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

$\frac{\partial f}{\partial x}(x,y) = y^3 - 2x$ $\qquad$ $\frac{\partial f}{\partial x}(1,2) = 2^3 - 2\cdot 1 = 6$

$\frac{\partial f}{\partial y}(x,y) = 3xy^2$ $\qquad$ $\frac{\partial f}{\partial y}(1,2) = 3\cdot 1\cdot 2^2 = 12$

$(D_{\underset{\sim}{u}} f)(1,2) = \frac{1}{2} \cdot 6 + \frac{\sqrt{3}}{2} \cdot 12$

$= 3 + 6\sqrt{3}$

2. $f(x,y) = x^2 \ln y$

$\frac{\partial f}{\partial x}(x,y) = 2x \ln y$ $\qquad$ $\frac{\partial f}{\partial y} = \frac{x^2}{y}$

$\frac{\partial f}{\partial x}(3,1) = 6 \cdot \ln 1$
$= \underline{\underline{0}}$

$\frac{\partial f}{\partial y}(3,1) = \frac{9}{1} = 9$

$\underset{\sim}{u} = \langle -\frac{5}{13}, \frac{12}{13} \rangle$

$(D_{\underset{\sim}{u}} f)(3,1) = -\frac{5}{13} \cdot \underline{\underline{0}} + \frac{12}{13} \cdot 9$

$= 0 + \cancel{108} \; \frac{108}{13}$

$\nabla f(x,y) = (2x \ln y)\hat{i} + \left(\frac{x^2}{y}\right)\hat{j}$

$$\nabla f(x,y) = \frac{\partial f}{\partial x}(x,y)\,\hat{\imath} + \frac{\partial f}{\partial y}(x,y)\,\hat{\jmath}$$

1) $f(x,y) = \dfrac{x}{y}$

$$\frac{\partial f}{\partial x} = \frac{1}{y} \qquad \frac{\partial f}{\partial y} = -\frac{x}{y^2}$$

$$\frac{\partial f}{\partial x}(2,1) = 1 \qquad \frac{\partial f}{\partial y}(2,1) = -2$$

$$\nabla f(2,1) = 1\hat{\imath} + (-2)\hat{\jmath}$$

2) $\nabla f(2,1) = 1\hat{\imath} + (-2)\hat{\jmath}$

$$\vec{u} = \frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}$$

$$D_{\vec{u}} f(2,1) = \left(1\hat{\imath} + (-2)\hat{\jmath}\right)\cdot\left(\frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}\right)$$

$$= \frac{3}{5} - \frac{8}{5} = -1$$

$$D_{\vec{u}} f(x_0, y_0) = \vec{\nabla} f(x_0, y_0) \cdot \vec{u}$$

<u>Maximum directional derivative</u>

occurs when $\vec{u}$ points in the

same direction as the gradient

$$D_{\vec{u}} f(x_0, y_0) = \vec{\nabla} f(x_0, y_0) \cdot \vec{u}$$

$$= |\vec{\nabla} f(x_0, y_0)| \, \cancelto{1}{|\vec{u}|} \cos\theta$$

Max. Derectional derivative =

$$|\vec{\nabla} f(x_0, y_0)|$$

$$f(x,y) = xe^{xy}$$

$$\frac{\partial f}{\partial x}(x,y) = e^{xy} + xye^{xy}$$

~~$\frac{\partial f}{\partial x}(0,2) =$~~

$$\frac{\partial f}{\partial y}(x,y) = xe^{xy}\cdot x = x^2e^{xy}$$

find: $\nabla f(0,2)$

max rate of change: $|\nabla f(0,2)|$

Direction of max rate of change:

Direction of $\nabla f(0,2)$

~~$\nabla f(0,2) = 3\hat{\imath}$~~

$$\frac{\partial f}{\partial x}(0,2) = e^0 + 0\cdot e^{\cdot\cdot} = 1$$

$$\frac{\partial f}{\partial y}(0,2) = 0$$

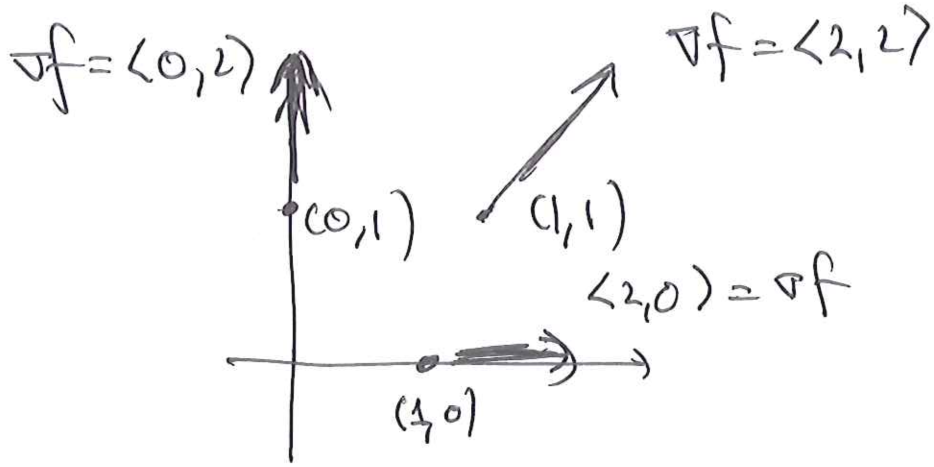
$\nabla f(0,2) = \hat{\imath}$

$|\nabla f(0,2)| = 1$

$\vec{u} = \hat{\imath}$

$$f(x,y) = x^2+y^2$$

$$\nabla f(x,y) = 2x\,\hat{i} + 2y\,\hat{j}$$



Level curves of $f(x,y) = x^2+y^2$

$$x^2+y^2 = k$$

So level curves are <u>circles</u>