Vectors,
or
How to Move Around in Space

Peter A. Perry

January 11, 2019

Homework

- Re-read section 12.2, pp. 798–804
- Begin work on problems 3-35 (odd), 41, 45, 47, pp. 805–807
- Continue working on Webwork A1 – Remember to access WebWork *only through Canvas!*

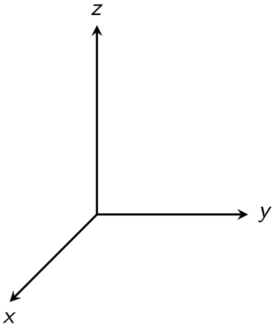
Also, read section 12.3, pp. 807–813 for Monday

Unit I: Geometry and Motion in Space

Goals of the Day

- Understand vectors as *displacements*
- Understand how to combine vectors by addition, subtraction, and scalar multiplication
- Understand *components* of vectors
- Understand *unit vectors*, and know the standard basis vectors **i**, **j**, and **k**
- Use vectors to solve problems involving forces and velocities

A **vector** is a set of instructions for how to move from one location in space to another. We've already seen this in our discussion of coordinate systems.



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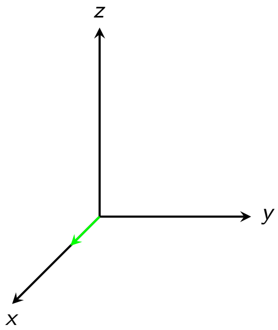

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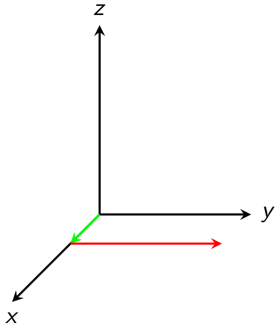

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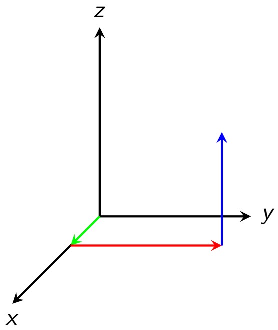

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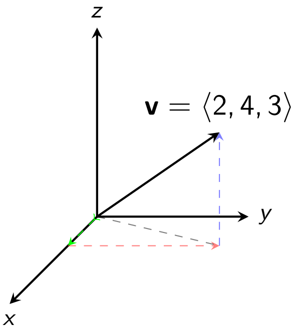

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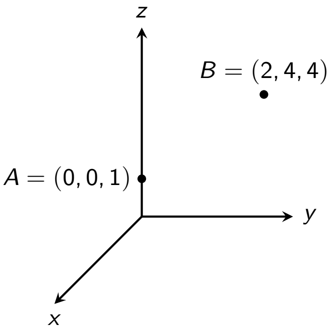
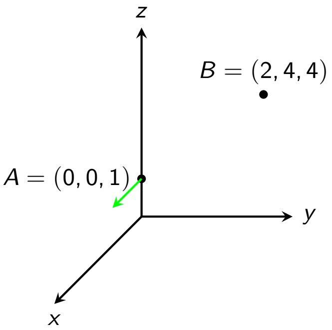
In this picture:

- the **initial point** of the vector is $(0, 0, 0)$
- the **final point** is $(2, 4, 3)$.

We could also choose a different initial point...

Let's begin at $A = (0, 0, 1)$ The vector $\mathbf{v} = \langle 2, 4, 3 \rangle$ is an instruction to move





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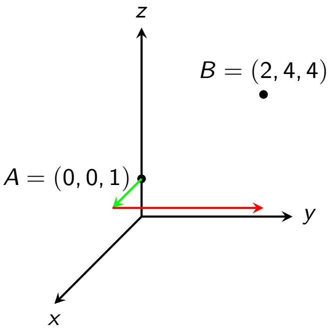

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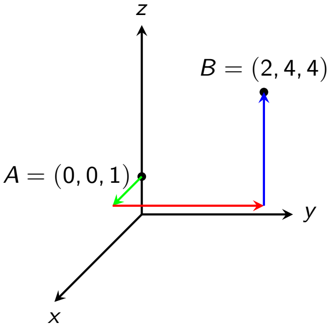

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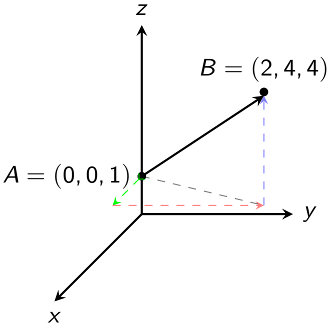

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In this picture:

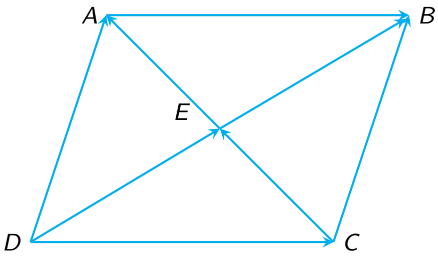
- the **initial point** of the vector is $A = (0, 0, 1)$
- the **final point** is $B = (2, 4, 4)$.

Another name for the vector $\mathbf{v}$ is

$$\overrightarrow{AB}.$$

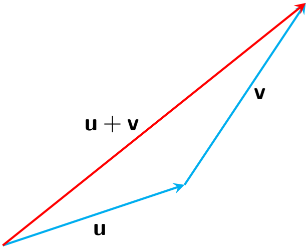
Puzzler

Can you name all of the equal vectors in the parallelogram shown below?



Vector Addition - Triangle Law

Vector Addition If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so that the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the sum $\mathbf{u}+\mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$



The Triangle Law

Vector Addition - Parallelogram Law

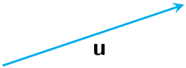
To add **u** and **v**, we can either:

The Parallelogram Law

Vector Addition - Parallelogram Law

To add **u** and **v**, we can either:

- Begin with **u**



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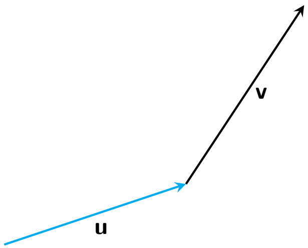

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To add **u** and **v**, we can either:

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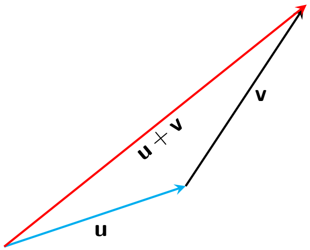
- Displace by **v**

Vector Addition - Parallelogram Law



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To add $\mathbf{u}$ and $\mathbf{v}$, we can either:

- Begin with $\mathbf{u}$
- Displace by $\mathbf{v}$
- Obtain $\mathbf{u} + \mathbf{v}$

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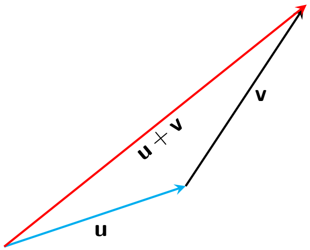

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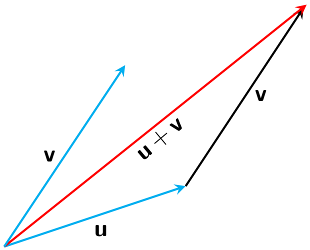

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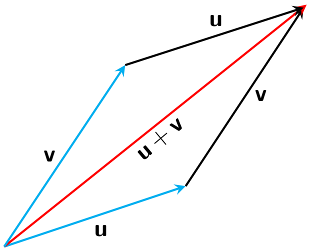

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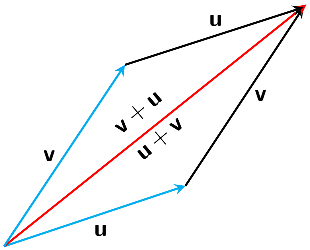

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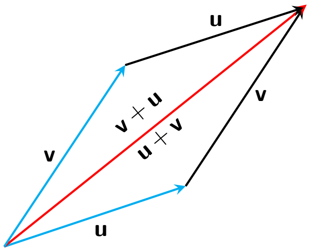

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Notice that

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

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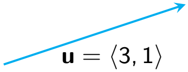

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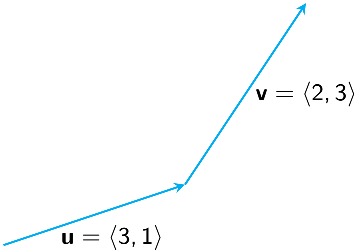

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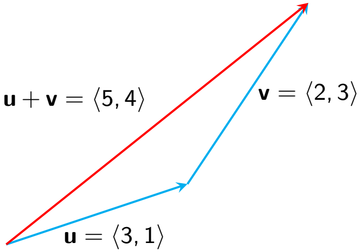
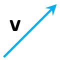


Scalar Multiplication

Scalar Multiplication If c is a scalar and $\mathbf{v}$ is a vector, then the **scalar multiple** $c\mathbf{v}$ is a vector $|c|$ times the length of $\mathbf{v}$ and whose direction is:



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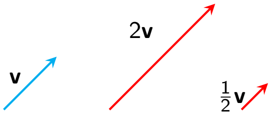

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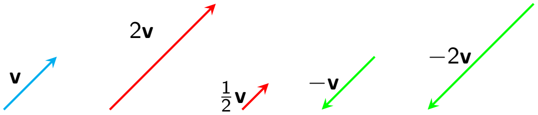

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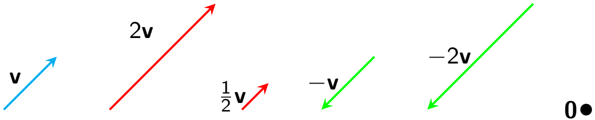
- The *zero vector* $\mathbf{0}$ if $c = 0$



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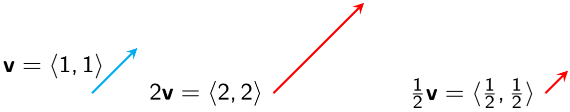

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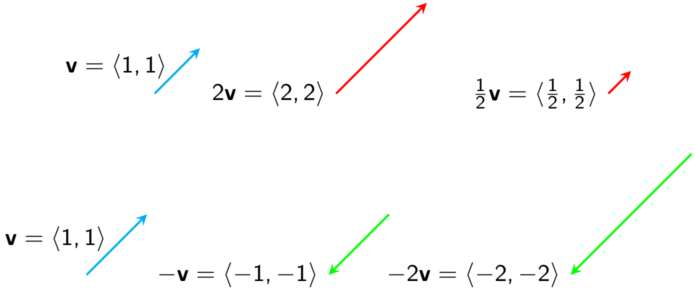
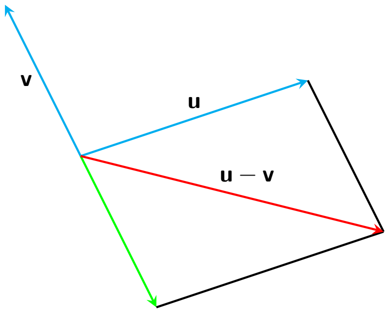


Vector Subtraction

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-1)\mathbf{v}$$



Vector Subtraction - Spoiler

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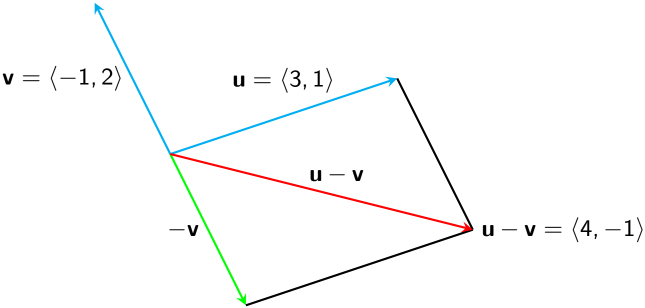

$$\langle 3, 1 \rangle - \langle -1, 2 \rangle = \langle 4, -1 \rangle$$

Vector Algebra

We've seen three operations on vectors: addition, scalar multiplication, and subtraction. Here are some basic rules for how these operations interact (see your text, p. 802, and know these properties!)

Properties of Vectors If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors, and c, d are scalars:

$$\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a} \qquad \mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$$

$$\mathbf{a}+\mathbf{0}=\mathbf{a} \qquad \mathbf{a}+(-\mathbf{a})=\mathbf{0}$$

$$c(\mathbf{a}+\mathbf{b})=c\mathbf{a}+c\mathbf{b} \qquad (c+d)\mathbf{a}=c\mathbf{a}+d\mathbf{a}$$

$$(cd)\mathbf{a}=c(d\mathbf{a}) \qquad 1\mathbf{a}=\mathbf{a}$$

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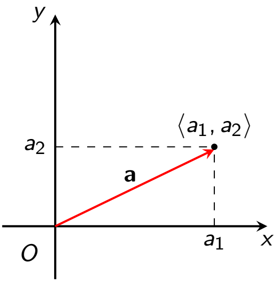

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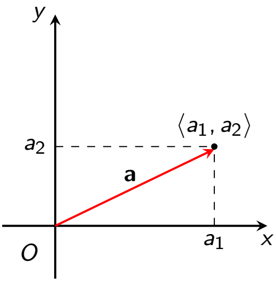

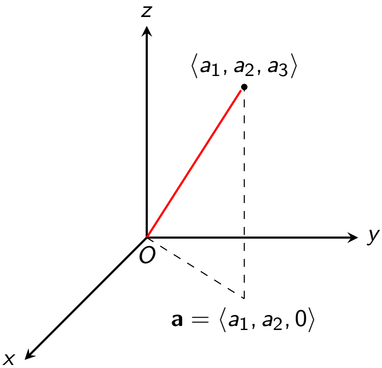

$$\mathbf{a} = \langle a_1, a_2 \rangle$$

$$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$$

Vector Operations in Components

- The vector $\overrightarrow{AB}$ from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ has components

$$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

- The **length** of a two-dimensional vector $\mathbf{a} = \langle a_1, a_2 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$

- The **length** of a three-dimensional vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Vector Operations in Components

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then:

$$\begin{aligned} \mathbf{a} + \mathbf{b} &= \langle a_1 + b_1, a_2 + b_2 \rangle \\ \mathbf{a} - \mathbf{b} &= \langle a_1 - b_1, a_2 - b_2 \rangle \\ c\mathbf{a} &= \langle ca_1, ca_2 \rangle \end{aligned}$$

What are the corresponding rules for three-dimensional vectors?

If $\mathbf{a} = \langle 2, 1, 2 \rangle$ and $\mathbf{b} = \langle 3, -1, 5 \rangle$, find:

- $\mathbf{a} - \mathbf{b}$
- $2\mathbf{a} + 3\mathbf{b}$
- $|\mathbf{a} - \mathbf{b}|$

Standard Basis Vectors

Every three-dimensional vector can be expressed in terms of the **standard basis vectors**

$$\mathbf{i} = \langle 1, 0, 0 \rangle, \quad \mathbf{j} = \langle 0, 1, 0 \rangle, \quad \mathbf{k} = \langle 0, 0, 1 \rangle$$

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, then another way of writing $\mathbf{a}$ is

$$a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$

The vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ have length 1. Any such vector is called a *unit vector*.

Unit Vectors

You can make any nonzero vector a unit vector if you scalar multiply by the inverse of its length.

Find a unit vector in the direction of the vector $\mathbf{i} + 2\mathbf{j}$

Find a unit vector in the direction of the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$

1. A quarterback throws a football with an angle of elevation of 40° and a speed of 60 ft/sec. Find the horizontal and vertical components of the velocity.
2. A crane suspends a 500 lb steel beam horizontally by support cables. Each support cable makes an angle of 60° with the beam. The cables can withstand a tension of up to 275 pounds. Would you feel safe standing below this rig?
3. A boatman wants to cross a canal that is 3 km wide and wants to land at a point 2 km upstream from his starting point. The current in the canal flows at 3.5 km/hr and the speed of his boat is 13 km/hr.
   (a) In what direction should he steer?
   (b) How long will the trip take?

Lecture Review

- We saw that vectors are *displacements* or instructions for moving from one point to another in the plane or in space
- We learned the operations of vector addition, vector subtraction, and scalar multiplication
- We learned how to express vectors in terms of *components*
- We learned about the unit vectors **i**, **j**, and **k** and how to form a *unit vector* from any nonzero vector **v**: multiply **v** by the recipricol of its length