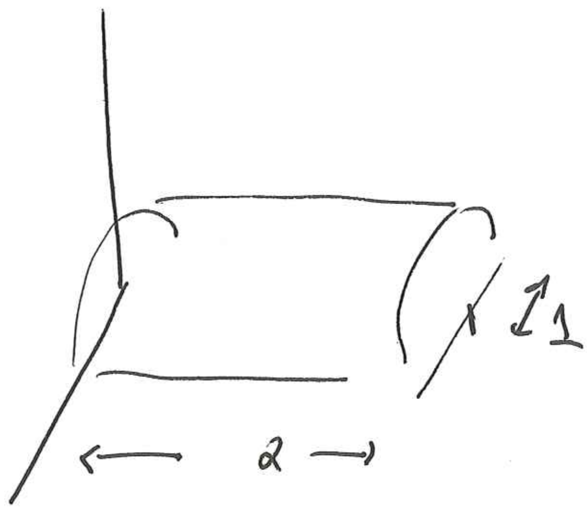
Double Integrals



$$V_{cy} = \frac{1}{4}\pi r^2 h$$

$$V_{\triangle} = \frac{1}{2}\pi r^2 h$$

$$r = 1$$

$$h = 2$$

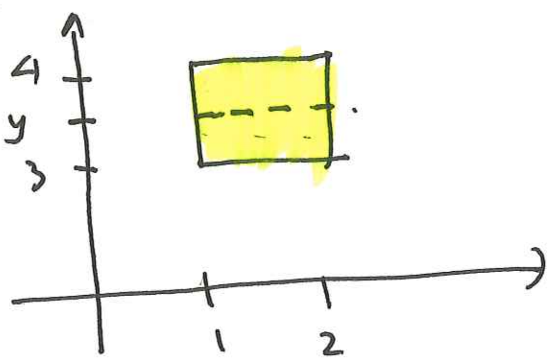
$$V_{\triangle} = \frac{1}{2}\pi \cdot 1^2 \cdot 2 = \pi$$

$$f(x,y) = x^2 y$$

Goal: Find

$$\boxed{\iint_D x^2 y \, dA}$$ Double integral

If D = the rectangle $\overset{x}{[1,2]} \times \overset{y}{[3,4]}$



$$\iint_D x^2 y \, dA = \cancel{\left(\int_1^2 x^2 y \, dx \right)}$$

$$= \boxed{\int_3^4 \left(\int_1^2 x^2 y \, dx \right) dy}$$

Iterated integral

$$= \int_3^4 \left(\left[y \cdot \frac{x^3}{3} \right] \Bigg|_{x=1}^{x=2} \right) dy$$

$$= \int_3^4 \left(y \cdot \left(\frac{8}{3} - \frac{1}{3} \right) \right) dy$$

$$= \int_3^4 \frac{7}{3} y \, dy$$

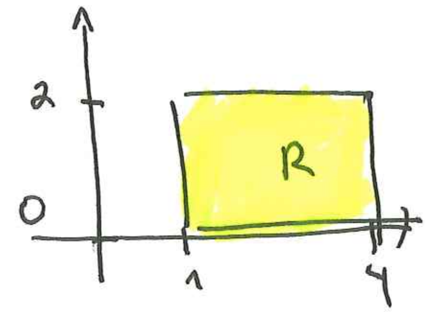
$$= \frac{7}{3} \frac{y^2}{2} \Bigg|_{y=3}^{y=4}$$

$$= \frac{7}{2 \cdot 3} \cdot (16 - 9)$$

$$= \frac{7}{2 \cdot 3} \cdot 7$$

$$= \frac{49}{2 \cdot 3} = \boxed{\frac{49}{6}}$$

$$\int_1^4 \underbrace{\int_0^2 (6x^2y-2x)\,dy}\,dx \quad \left(= \iint_R (6x^2y-2x)\,dA\right)$$



① $$\int_0^2 (6x^2y-2x)\,dy =$$

$$\left[6\frac{x^2y^2}{2} - 2xy\right]\Bigg|_{y=0}^{y=2} =$$

$$\left[6\frac{x^2\cdot 2^2}{2} - 2x\cdot 2\right] =$$

$$[12x^2 - 4x]$$

② $$\int_1^4 (12x^2-4x)\,dx = \left[12\cdot\frac{x^3}{3} - 2x^2\right]\Bigg|_1^4$$

$$= \left[12\cdot\frac{64}{3} - 2\cdot 16\right] - \left[12\cdot\frac{1}{3} - 2\right]$$

2) $$\underbrace{\int_1^3 \left(\underbrace{\int_1^5 \frac{1}{x} \cdot \frac{\ln y}{y}\, dy}_{1} \right) dx}_{2}$$

(1) $$\int_1^5 \frac{1}{x} \frac{\ln y}{y}\, dy =$$

$du = dy/y$

$u = \ln y$

$y = 1 \quad u = \ln 1 = 0$

$y = 5 \quad u = \ln 5$

$$\frac{1}{x} \int_0^{\ln 5} u \, du =$$

$$\frac{1}{x} \left[\frac{u^2}{2}\right]\Big|_0^{\ln 5} =$$

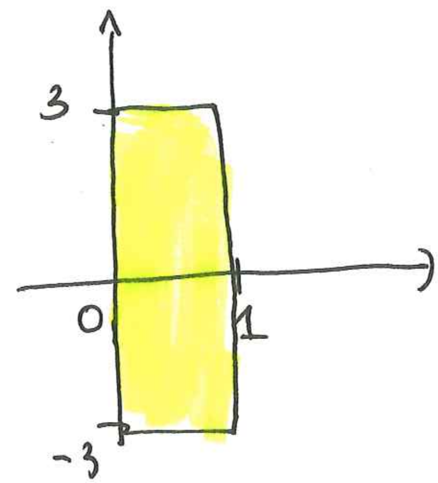
$$\frac{1}{x} \frac{1}{2} (\ln 5)^2$$

(2) $$\int_1^3 \frac{1}{x} \frac{1}{2} (\ln 5)^2 \, dx =$$

$$\frac{1}{2} (\ln 5)^2 \int_1^3 \frac{1}{x}\, dx =$$

$$\frac{1}{2} (\ln 5)^2 \left[\ln x \Big|_{x=1}^{x=3} \right] = \underline{\frac{1}{2} (\ln 5)^2 \ln 3}$$

2) $\displaystyle\iint_R \frac{xy^2}{x^2+1}\,dA \qquad R = \overset{x}{[0,1]} \times \overset{y}{[-3,3]}$



$$= \int_{-3}^{3} \left(\int_0^1 \frac{xy^2}{x^2+1}\,dx \right) dy$$

$$= \int_{-3}^{3} y^2 \left(\int_0^1 \frac{x}{x^2+1}\,dx \right) dy$$

$u = x^2+1$

$du = 2x\,dx$

$x = 0 \to u = 1$

$x = 1 \quad u = 2$

$$= \int_{-3}^{3} y^2 \left(\int_1^2 \frac{1}{u} \cdot \frac{1}{2}\,du \right) dy$$

$$= \int_{-3}^{3} y^2 \left[\frac{1}{2} \ln(u) \Big|_1^2 \right] dy$$

$$= \int_{-3}^{3} y^2 \, \frac{1}{2} \ln(2) \, dy$$

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1) $4x + 6y - 12z + 15 = 0$

above $\overset{x}{[-1,2]} \times \overset{y}{[-1,1]} = R$

volume under graph of f $\quad \iint_R f(x,y)\, dA$

$$12z = 4x + 6y + 15$$

$$z = \frac{1}{3}x + \frac{1}{2}y + \frac{5}{4} = f(x,y)$$

$$V = \iint_{[-1,2]\times[-1,1]} \left(\frac{1}{3}x + \frac{1}{2}y + \frac{5}{4}\right) dA$$

$$\int_1^3 \left(\int_{x/3}^{4x/3} (\underset{+}{x} + \underset{\#}{3y})\, dy \right) dx =$$

$$= \int_1^3 \left\{ \left[\underset{+}{xy} + \underset{\#}{\frac{3y^2}{2}} \right] \Bigg|_{y=x/3}^{y=4x/3} \right\} dx =$$

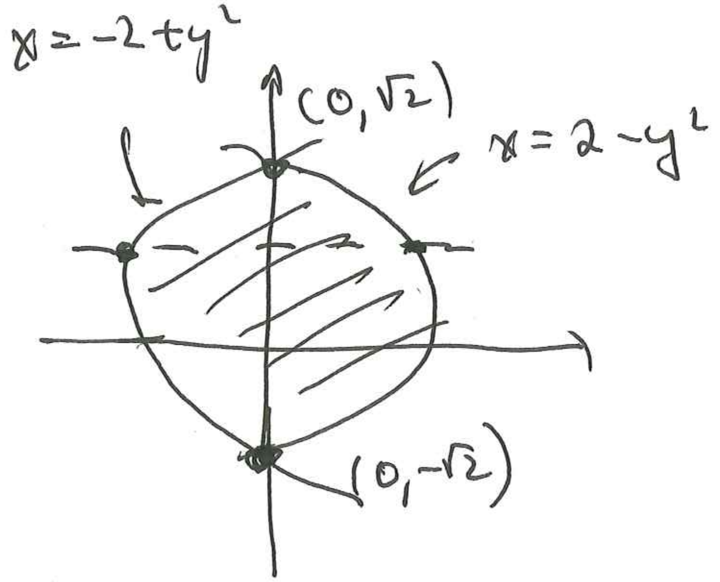
$$= \int_1^3 \left\{ \underset{+}{\overset{①}{x \cdot \frac{4x}{3}}} + \underset{\#}{\overset{②}{\frac{3}{2}\left(\frac{4x}{3}\right)^2}} - \left[\underset{+}{\overset{③}{x \cdot \frac{x}{3}}} + \underset{\#}{\overset{④}{\frac{3}{2}\left(\frac{x}{3}\right)^2}} \right] \right\} dx$$

$$= \int_1^3 \left\{ \overset{①}{\frac{4x^2}{3}} + \overset{②}{\frac{\cancel{3}}{2} \cdot \frac{16x^2}{\cancel{9}\,3}} - \left[\overset{③}{\frac{x^2}{3}} + \frac{3}{2}\,\frac{x^2}{9} \right] \right\} dx$$

$$= \int_1^3 \underset{①-③}{x^2} + \underset{②-④}{\left(\frac{16}{6}x^2 - \frac{\cancel{3}\,1}{\cancel{18}\,6}x^2 \right)} dx$$

$$= \int_1^3 \left(x^2 + \frac{15}{6}x^2\right) dx$$

$$= \int_1^3 \frac{21}{6} x^2 \, dx$$



$$2 - y^2 = -2 + y^2$$
$$4 = 2y^2$$
$$y = \pm\sqrt{2}$$

$$\iint_D 1 \, dA = \int_{-\sqrt{2}}^{\sqrt{2}} \left(\int_{-2+y^2}^{2-y^2} 1 \, dx \right) dy$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \left[[x] \Big|_{-2+y^2}^{2-y^2} \right] dy$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \left[2 - y^2 - (-2 + y^2) \right] dy$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} (4 - 2y^2) \, dy$$

$$= \left[4y - \frac{2}{3} y^3 \right] \Big|_{-\sqrt{2}}^{\sqrt{2}}$$