Double Integral

$$\iint_D f(x,y)\, dA$$

$\uparrow$ region in xy-plane

Iterated Integral

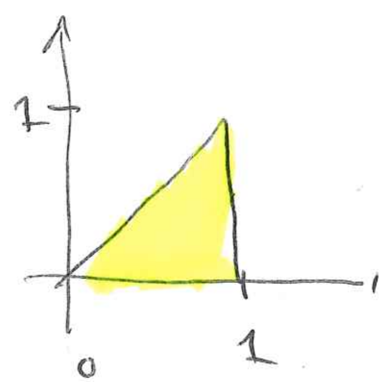
$$\int_0^1 \int_2^3 x^2 y \, dx\, dy$$

$f(x,y) = x^2 y$

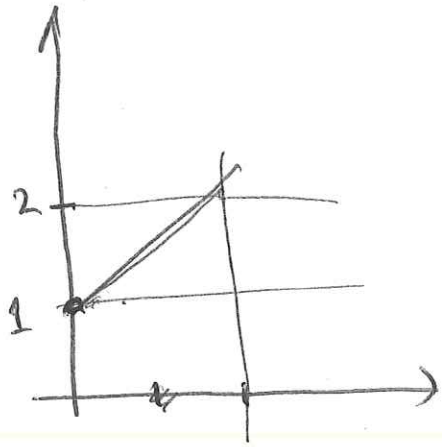
$D = [2,3] \times [0,1]$

$\Downarrow x$ $\quad \Downarrow y$

$$\int_0^1 \left(\int_0^x x \cos y \; dy \right) dx$$



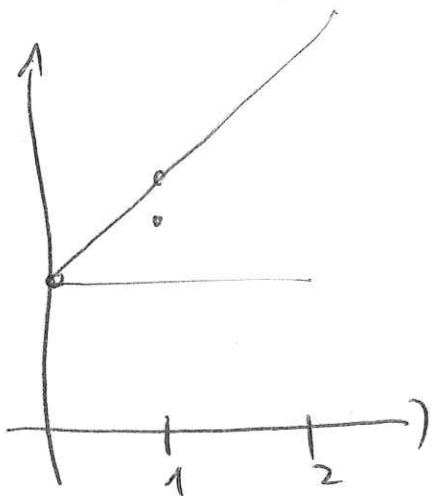
$D = \{(x,y) : 1 \leq y \leq 2,$
$y-1 \leq x \leq 1$



$x \geq y-1$

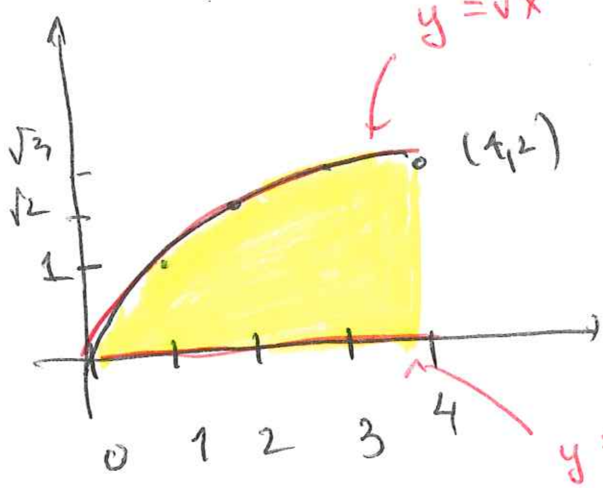
$x+1 \geq y$

Draw $y = x+1$



$$D = \{(x,y):\ 0 \le x \le 4,\quad 0 \le y \le \sqrt{x}\}$$

$y = \sqrt{x}$ $\quad g_2(x) = \sqrt{x}$ (top)



$y = 0$ $\quad g_1(x) = 0$ (bottom)

$$\iint_D \frac{y}{x^2+1}\, dA =$$

$$\int_0^4 \left(\int_0^{\sqrt{x}} \frac{y}{x^2+1}\, dy \right) dx =$$

$$\int_0^4 \left(\left[\frac{1}{x^2+1} \frac{y^2}{2} \right]_{y=0}^{y=\sqrt{x}} \right) dx =$$

$$\int_0^4 \left(\frac{1}{x^2+1} \cdot \frac{x}{2} \right) dx =$$

4

$$\frac{1}{2}\int_0^4 \frac{x}{x^2+1}\,dx =$$

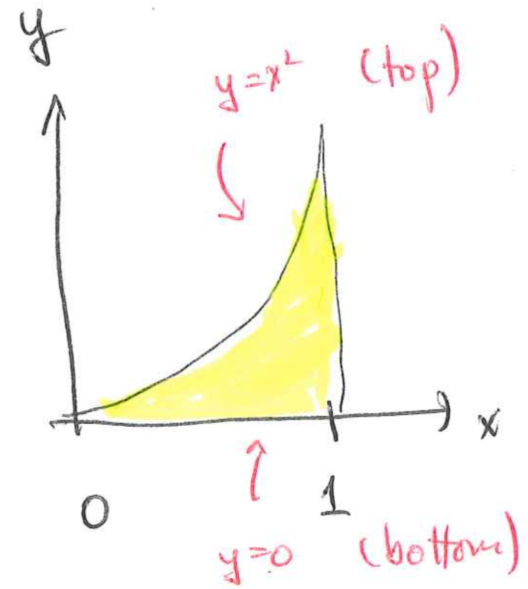
$$\frac{1}{4}\int_0^4 \frac{2x}{x^2+1}\,dx =$$

$u = x^2+1$
$du = 2x\,dx$
$x=0 \quad u=1$
$x=4 \quad u=17$

$$\frac{1}{4}\int_1^{17} \frac{du}{u} =$$

$$\frac{1}{4}(\ln u)\Big|_1^{17} =$$

$$\frac{1}{4}\ln 17$$



$$\iint_R x\cos y \, dA = \int_?^? \left(\int_?^? x\cos y \, dy \right) dx$$

$$= \int_0^1 \left(\int_0^{x^2} x\cos y \, dy \right) dx$$

$du = 2x\,dx$

$u = x^2$

$x=0 \quad u=0$

$x=1 \quad u=1$

$$= \int_0^1 \left[\left[x \sin y \right]_{y=0}^{y=x^2} \right] dx$$

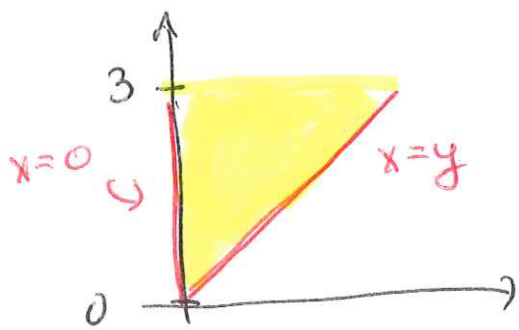
$$= \int_0^1 \left[x \sin(x^2) \right] dx$$

$$= \frac{1}{2} \int_0^1 \sin(u) \, du$$

$$= \frac{1}{2} \left(-\cos(u) \right) \Big|_{u=0}^{u=1}$$

6

$$D = \{(x,y) : 0 \le y \le 3, \quad \underset{h_1(y)}{0} \le x \le \underset{h_2(y)}{y}\}$$



$$\iint_D e^{-y^2}\,dA = \int_0^3 \left(\int_0^y e^{-y^2}\,dx \right) dy$$

$$= \int_0^3 \left(\left[x e^{-y^2} \Big|_{x=0}^{x=y} \right] \right) dy$$

$$= \int_0^3 y e^{-y^2}\,dy$$

$u = y^2$

$y=0 \qquad u=0$

$y=3 \qquad u=9$

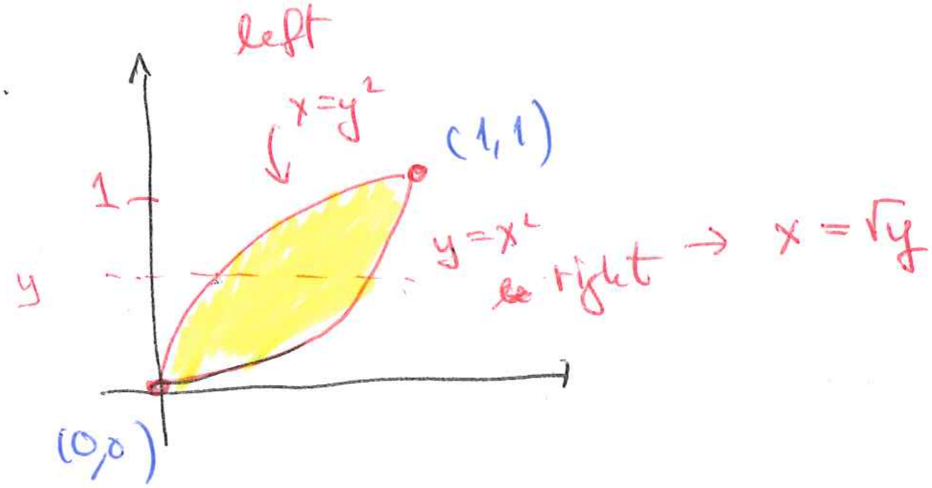
$du = 2y\,dy$

$$= \frac{1}{2}\int_0^9 e^{-u}\,du$$

$$= \frac{1}{2}\left(-e^{-u}\right)\Big|_{u=0}^{u=9}$$

$$= \frac{1}{2}(1-e^{-9})$$

$$3x+3y-z=0 \quad \Rightarrow \quad z=3x+3y$$



$$\iint_D (3x+3y)\, dA =$$

$$\int_{\cancel{?}\;0}^{\cancel{?}\;1}\left[\int_{?\;y^2}^{?\;\sqrt{y}}\left(3x+3y\right) dx\right] dy$$

$$= \int_0^1\left(\left[\int_{y^2}^{\sqrt{y}} (3x+3y)\right] dx\right) dy$$

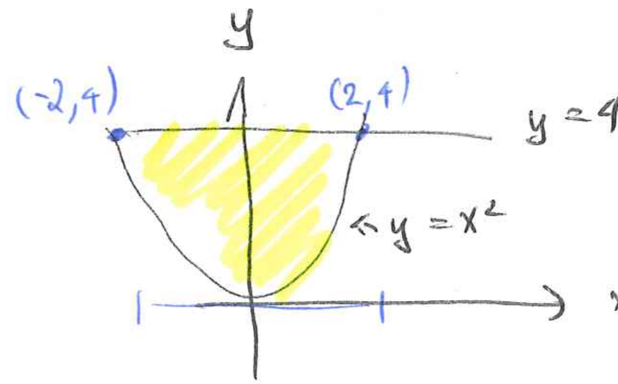
Find the volume enclosed by

$z = x^2$

$y = x^2$

$z = 0$

$y = 4$



~~y's range from 0 to 4~~

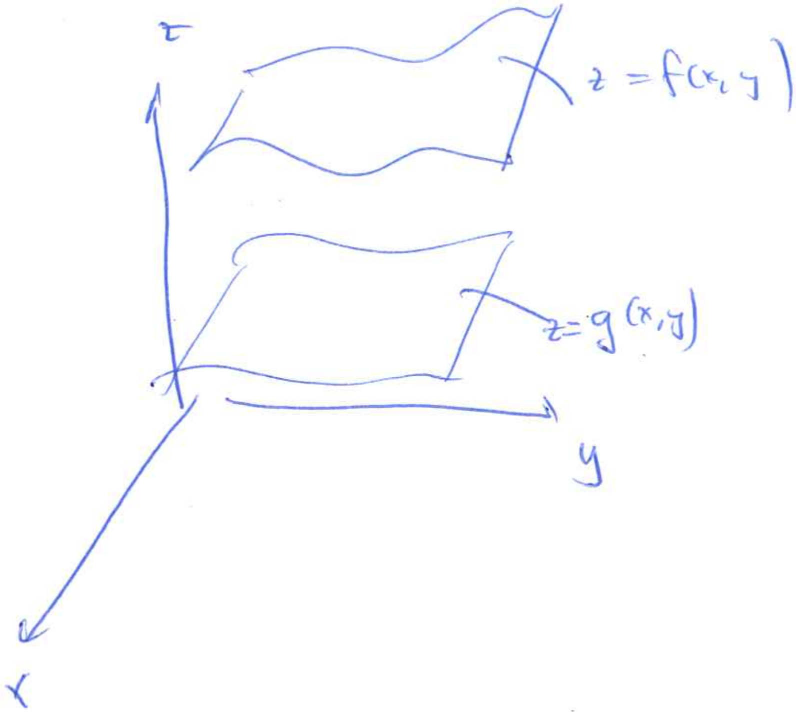
$z = x^2$ sits over the xy plane in a region bounded $y = 4$ and $y = x^2$

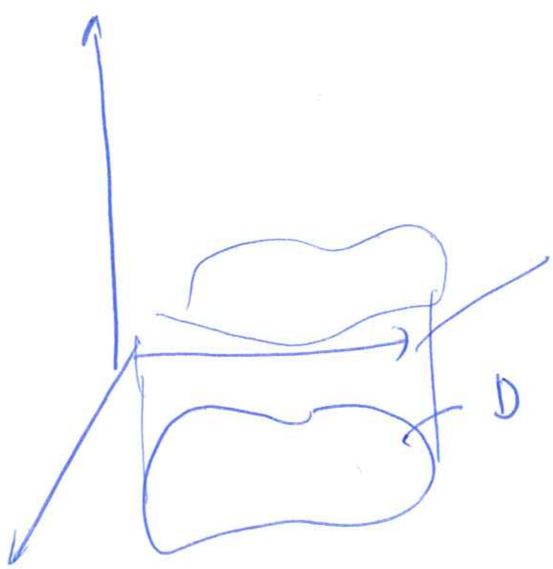
$$\iint_D f(x,y)\, dA =$$

$f(x,y) = x^2$

$D: \{-2 \le x \le 2,\ 0 \le y \le x^2\}$

$$\Rightarrow \text{Volume} = \boxed{\int_{-2}^{2} \left(\int_0^{x^2} (x^2)\, dy \right) dx}$$





$$\int_D 1 \, dA = \text{area}(D)$$

Find the volume enclosed by

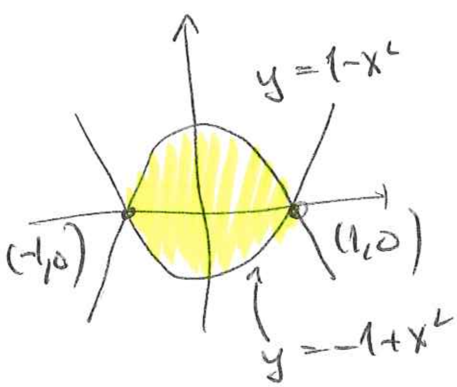
$$y = 1 - x^2$$

$$y = x^2 - 1$$

$$x + y + z = 2$$

$$2x + 2y - z + 10 = 0$$

1) Region D in xy plane (corrected from class)



So: $-1 \leq x \leq 1$

$-1 + x^2 \leq y \leq 1 - x^2$

2) Height functions

$z = 2 - y - x$ (this plane lies below)

$z = 10 + 2x + 2y$ (this plane lies above)

$$\boxed{V = \int_{-1}^{1} \int_{-1+x^2}^{1-x^2} \left[(10 + 2x + 2y) - (2 - y - x) \right] dy\, dx}$$