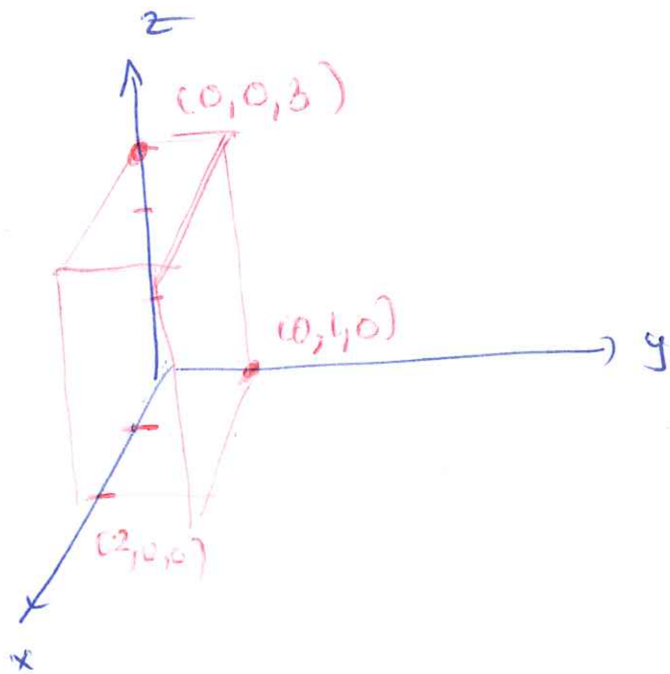
3/8/19 ①

$$f(x,y,z) = xy + z^2$$

$$B = \{(x,y,z) : 0 \le x \le 2,\ 0 \le y \le 1,\ 0 \le z \le 3\}$$

$$\iiint_B f(x,y,z)\, dV$$



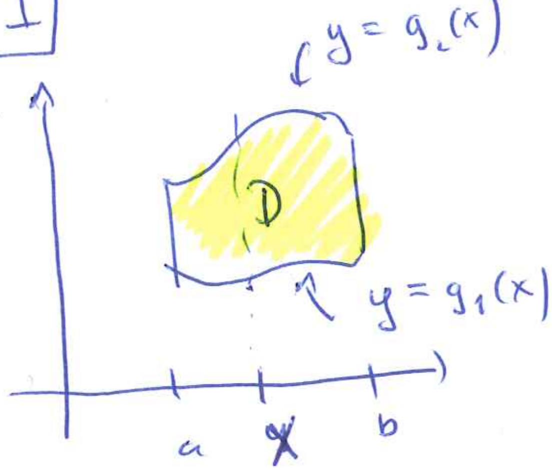
$$\iiint_B f(x,y,z)\, dV =$$

$$\int_0^3 \int_0^1 \int_0^2 (xy + z^2)\, dx\, dy\, dz =$$

$$\int_0^3 \int_0^1 \left(\int_0^2 (xy + z^2)\, dx \right) dy\, dz =$$

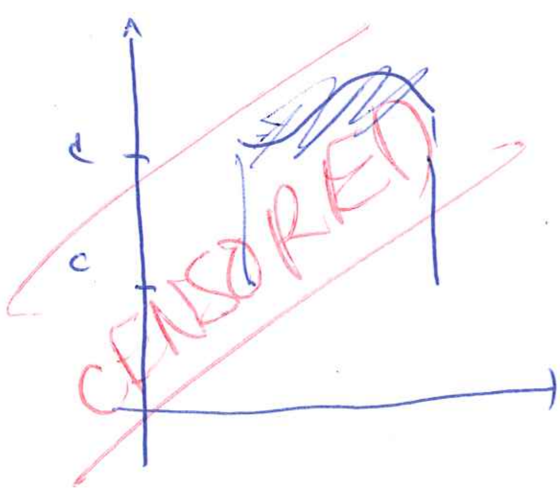
Reminder: Double Integrals:

Type I

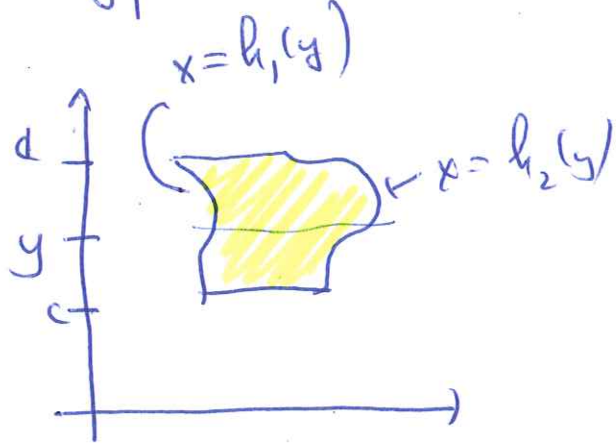


$$\iint_D f(x,y)\, dA = \int_a^b \left(\int_{g_1(x)}^{g_2(x)} f(x,y)\, dy \right) dx$$

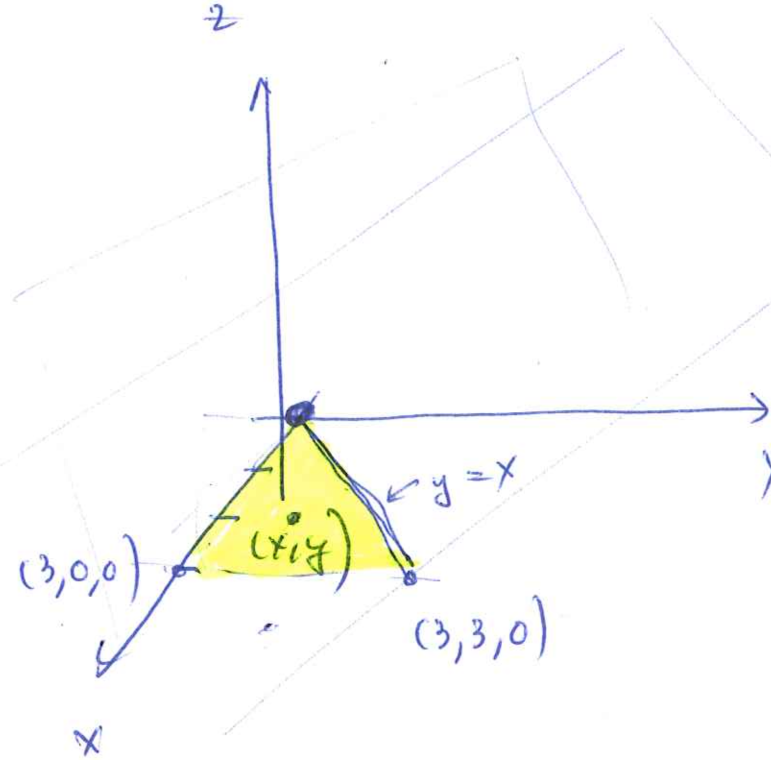
Type II



Type II



$$\iint_D f(x,y)\, dA = \int_c^d \left(\int_{h_1(y)}^{h_2(y)} f(x,y)\, dx \right) dy$$



D.

$E = \{ (x,y,z) :$

$0 \le x \le 3,$

$0 \le y \le x,$

$x-y \le z \le x+y \}$

$z = x - y$

$x - y - z = 0$

$\langle 1, -1, -1 \rangle$

$z = x + y$

$x + y - z = 0$

$\langle 1, 1, -1 \rangle$

$$\iiint_E y \, dV =$$

$$\int_0^3 \left(\int_0^x \left(\int_{x-y}^{x+y} y \, dz \right) dy \right) dx =$$

$$\int_0^2 \left(\int_0^x \left(yz \Big|_{z=x-y}^{z=x+y} \right) dy \right) dx =$$

$$\int_0^2 \left(\int_0^x \left(y(x+y) - y(x-y) \right) dy \right) dx = \boxed{3/8/19}$$ (5)

$$\int_0^2 \int_0^x \left(\left[y\cancel{x} + y^2 - y\cancel{x} + y^2 \right] \right) dy \, dx =$$

$$\int_0^2 \int_0^x 2y^2 \, dy \, dx =$$

$$\int_0^2 \left\{ \left[\frac{2y^3}{3} \right]_{y=0}^{y=x} \right\} dx =$$

$$\int_0^2 \frac{2x^3}{3} \, dx =$$

$$\left[\frac{2}{3} \cdot \frac{x^4}{4} \right]_{x=0}^{x=2} =$$

$$\frac{2}{3} \cdot \frac{2^4}{4} = \frac{32}{12} = \frac{8}{3}$$

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<u>Iterated Integrals</u>

1) $$\int_0^1 \left(\int_y^{2y} \left(\int_0^{x+y} 6xy \; dz \right) dx \right) dy$$

$$= \int_0^1 \left(\int_y^{2y} \left(\left[6xyz \right]_{z=0}^{z=x+y} \right) dx \right) dy$$

$$= \int_0^1 \left(\int_y^{2y} \left[6xy(x+y) - 0 \right] dx \right) dy$$

$$= \int_0^1 \left(\int_y^{2y} \left(6x^2y + 6xy^2 \right) dx \right) dy$$

$$= \int_0^1 \left[2x^3y + 3x^2y^2 \right]_{x=y}^{x=2y} dy$$

$$= \int_0^1 \left[2(2y)^3 \cdot y + 3 \cdot (2y)^2 \cdot y^2 - 2y^4 + 3y^4 \right] dy$$

$$\int_0^1 \left(\int_0^1 \left(\int_0^{\sqrt{1-z^2}} \frac{z}{y+1} \, dx \right) dz \right) dy =$$

$$\int_0^1 \left(\int_0^1 \left(\frac{z}{y+1} x \Big|_{x=0}^{x=\sqrt{1-z^2}} \right) dz \right) dy =$$

$$\int_0^1 \left(\int_0^1 \frac{z\sqrt{1-z^2}}{y+1} \, dz \right) dy =$$

$$\int_0^1 \frac{1}{y+1} \underbrace{\left(\int_0^1 z\sqrt{1-z^2} \, dz \right)}_{1/3} dy =$$

$u = 1-z^2$

$du = -2z \, dz$

$$\int_0^1 z\sqrt{1-z^2} \, dz = \int_1^0 -\frac{1}{2}\sqrt{u} \, du$$

$$= \frac{1}{2}\int_0^1 \sqrt{u} \, du$$

$$= \frac{1}{\cancel{2}} \frac{\cancel{2}}{3} u^{3/2} \Big|_0^1 = \frac{1}{3}$$

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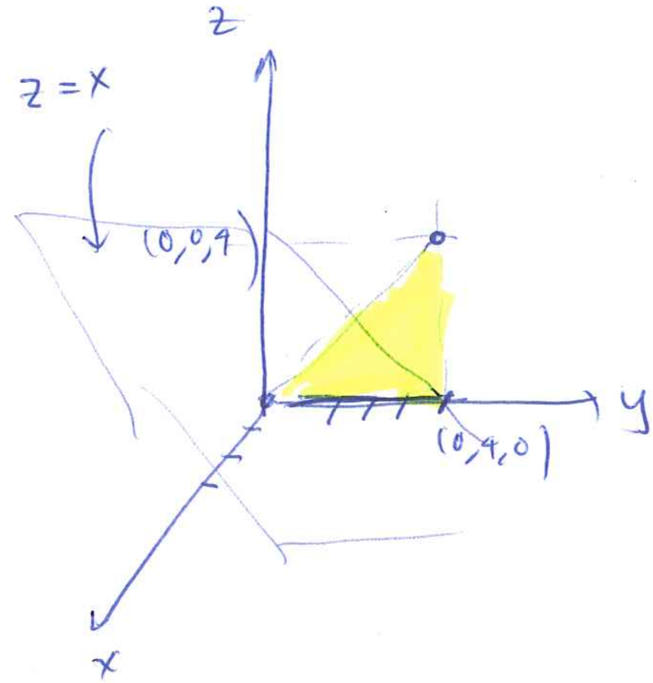
$$\int_0^1 \frac{1}{y+1} \cdot \frac{1}{3}\, dy =$$

$$\frac{1}{3} \ln(y+1) \Big|_0^1 =$$

$$\frac{1}{3}(\ln 2 - \ln 1) = \frac{1}{3}\ln 2$$

3/8/19 (9)



$$\iiint_E \frac{z}{x^2+y^2}\, dV =$$

$$\int_0^4 \left[\int_y^4 \left(\int_0^z \frac{z}{x^2+y^2} \, dx \right) dz \right] dy$$