Math 213 - Triple Integrals

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Homework

- Webwork B8 on 15.1-15.2 is due today, Friday March 8
- Webwork C1 on 15.3 is due today, Friday March 8
- Webwork C2 on 15.6 will due on Wednesday, March 20
- Practice problems for 15.6 are 3-21 (odd), 33, 37-45 (odd)
- You will have a quiz on 15.3 and 15.6 on the Thursday after Spring Break

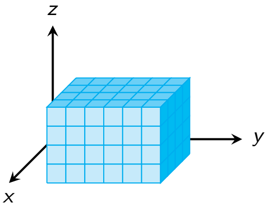
Unit III: Integral Calculus, Vector Fields

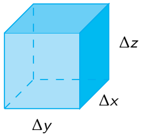
Lecture 24	Triple Integrals
Lecture 25	Triple Integrals, Continued
Lecture 26	Triple Integrals - Cylindrical Coordinates
Lecture 27	Triple Integrals - Spherical Coordinates
Lecture 28	Change of Variables for Multiple Integrals, I
Lecture 29	Change of Variable for Multiple Integrals, II
Lecture 30	Vector Fields
Lecture 31	Line Integrals (Scalar Functions)
Lecture 32	Line Integrals (Vector Functions)
Lecture 33	Fundamental Theorem for Line Integrals
Lecture 34	Green's Theorem
Lecture 35	Exam III Review

Goals of the Day

- Understand triple integrals as a limit of Riemann sums
- Understand how to compute triple integrals as iterated integrals
- Understand how to compute triple integrals over fiendishly contrived regions

Riemann Sums





Given a rectangular box

$$B = [a, b] \times [c, d] \times [r, s]$$

and a function $f(x, y, z)$, we can divide the box into cubes of side Δx, Δy, Δz and volume

$$\Delta V = \Delta x \, \Delta y \, \Delta z$$

The *triple integral* of f over the box B is the limit of Riemann sums

$$\sum_{i,j,k} f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*) \Delta V$$

and is denoted

$$\iiint_B f(x, y, z) \, dV$$

Triple Integrals as Iterated Integrals

If $B = [a, b] \times [c, d] \times [r, s]$ then

$$\iiint_B f(x, y, z)\, dV = \int_r^s \int_c^d \int_a^b f(x, y, z)\, dx\, dy\, dz$$

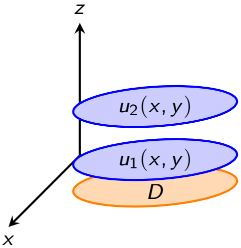
Evaluate $\iiint_B (xy + z^2)\, dV$ if

$$B = \{(x, y, z) : 0 \le x \le 2,\ 0 \le y \le 1,\ 0 \le z \le 3\}$$

Integrals Over Regions: Type I

Suppose that

$$E = \{(x, y, z) : (x, y) \in D,\ u_1(x, y) \le z \le u_2(x, y)\}.$$



$$\iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x, y, z)\, dz \right] dA$$

Find $\iiint_E y\, dV$ if E is the region over

$$D = \{0 \le x \le 3,\ 0 \le y \le x\}$$

where for each (x, y),

$$x - y \le z \le x + y$$

Practice with Iterated Integrals

1. Find $\int_0^1 \int_y^{2y} \int_0^{x+y} 6xy \, dz \, dx \, dy$
2. Find $\int_0^1 \int_0^1 \int_0^{\sqrt{1-z^2}} \frac{z}{y+1} \, dx \, dz \, dy$

Integrals over Regions: Type II

If

$$E = \{(x, y, z) : (y, z) \in D,\ u_1(y, z) \le x \le u_2(y, z)\}$$

then

$$\iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(y,z)}^{u_2(y,z)} f(x, y, z)\, dx \right] dA$$

Find $\displaystyle\iiint_E \frac{z}{x^2 + z^2}\, dV$ if

$$E = \{(x, y, z) : 1 \le y \le 4,\ y \le z \le 4,\ 0 \le x \le z\}.$$

Integrals over Regions: Type III

If

$$E = \{(x, y, z) : (x, z) \in D,\ u_1(x, z) \leq y \leq u_2(x, z)\}$$

then

$$\iiint_E f(x, y, z)\, dV = \iint_D \left[\int_{u_1(x,z)}^{u_2(x,z)} f(x, y, z)\, dy \right] dA$$

Find $\iiint_E \sqrt{x^2 + z^2}\, dV$ if E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$

Enjoy Spring Break!