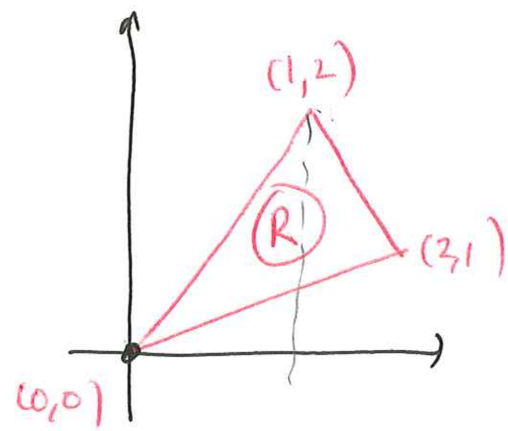
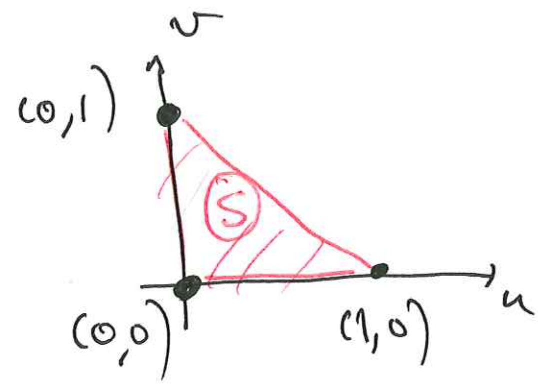
Find: $\iint_{R} (x-3y)\, dA$




① $x = 2u + v$

② $y = u + 2v$

solve for u and v in terms of x and y




$2\cdot①$ $2x = 4u + 2v$

$-②$ $y = u + 2v$

$2x - y = 3u$

$u = \frac{1}{3}(2x-y)$ — (1)

(−) ① $-x = -2u - v$

$2\cdot②$ $2y = 2u + 4v$

$2y - x = 3v$

$v = \frac{1}{3}(2y-x)$ — (2)

| x | y | u | v |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2 | 0 | 1 |
| 2 | 1 | 1 | 0 |

Jacobian Determinat

$$x = 2u + v$$

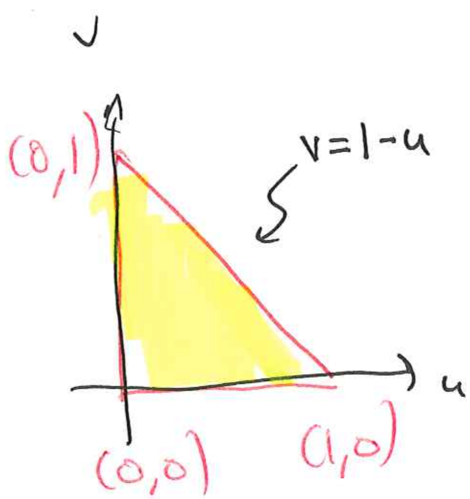
$$y = u + 2v$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$$

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = 3$$

$$\iint_R (x - 3y)\, dA = \iint_S [\overbrace{(2u+v)}^{x} - \overbrace{3(u+2v)}^{3y}] \cdot \underline{3} \cdot du\,dv$$

$$= \int_0^1 \left( \int_0^{1-u} (-u - 5v)\, 3\, dv \right) du$$



$u = x - y$

$v = x + y$

$S' = \{ (u, v) : \quad 0 \le u \le 2, \quad 0 \le v \le 3 \}$

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$$

$$\iint_R \underline{(x+y)} e^{x^2-y^2} \, dA$$

$$= \iint_S \underline{v} \; e^{uv} \cdot \underline{\underline{2}} \; du\,dv$$

$$= \int_0^2 \left( \int_0^3 v e^{uv} \;\; 2 \; dv \right) du$$

Cylindrical Coordinate:

$$\iiint_R f(x, y, z)\, dV =$$

$$\int \int \int f(r\cos\theta, r\sin\theta, z)\, dz\, \boxed{r}\, dr\, d\theta$$

Spherical Coordinates:

$$\iiint_R f(x, y, z)\, dV$$

$$\int \int \int f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)$$
$$\boxed{\rho^2 \sin\phi}\, d\rho\, d\phi\, d\theta$$

## cylindrical coordinate

$r \quad \theta \quad z$

$u \quad v \quad w$

$$x = u \cos v$$

$$y = u \sin v$$

$$z = w$$

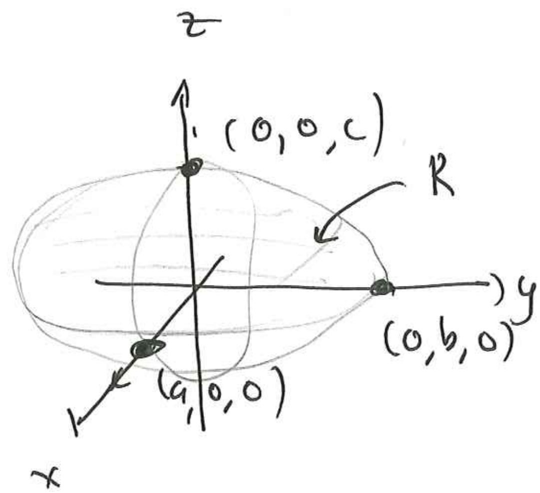
$$\begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \begin{vmatrix} \cos v & -u \sin v & 0 \\ \sin v & u \cos v & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \cos v \,(u \cos v) + u \sin v \sin v + 0$$

$$= u \cos^2 v + u \sin^2 v$$

$$= \boxed{u}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

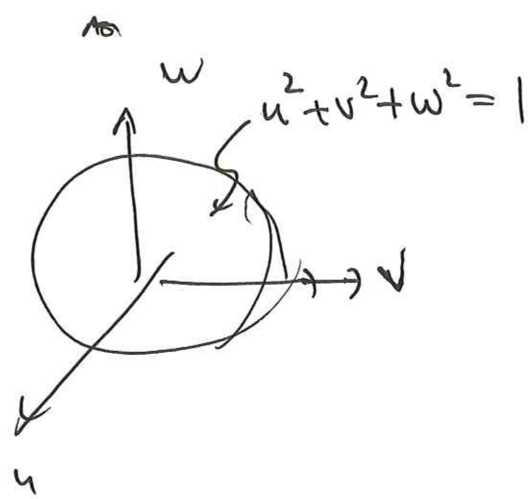



$$V = \iiint_R 1 \, dV$$

$$x = au \Rightarrow \frac{x}{a} = u$$

$$y = bv \Rightarrow \frac{y}{b} = v$$

$$z = cw \Rightarrow \frac{z}{c} = w$$




$$u^2 + v^2 + w^2 = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

$$V \equiv \iiint_{S'} 1 \cdot \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| \, du\, dv\, dw$$

$x = au$

$y = bv$

$z = cw$

$$\frac{\partial(x,y,t)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$$

$$= abc$$

$$V = \iiint_R 1 \, dV = \iiint_S 1 \cdot (abc) \, du\, dv\, dw$$

$$= (abc) \iiint_S 1 \cdot du\, dv \cdot dw$$

$$= (abc) \left( \frac{4}{3} \pi 1^3 \right)$$

$$= \boxed{\frac{4\pi}{3} abc}$$