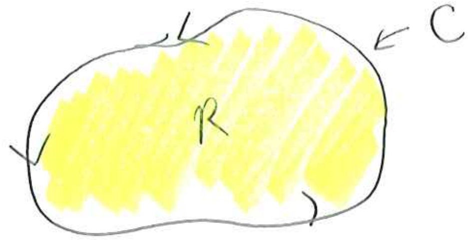


$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_C P\,dx + Q\,dy$$

$$= \oint_C \vec{F} \cdot d\vec{r}$$

if $\vec{F} = P(x,y)\hat{\imath} + Q(x,y)\hat{\jmath}$

$$\oint_C x^2\,dy - xy^2\,dy = \oint_C P\,dx + Q\,dy$$

$$P = x^2 \qquad Q = -xy^2$$

$$\frac{\partial P}{\partial y} = 0 \qquad \frac{\partial Q}{\partial x} = -y^2$$

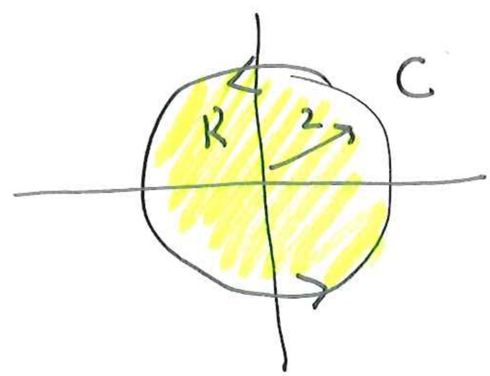
$$\oint_C x^2 y\,dx - xy^2\,dy =$$

$$\oint_C P\,dx + Q\,dy = \iint_R (-y^2 - x^2)\,dA$$

$$P(x,y) = x^2 y$$

$$Q(x,y) = -xy^2$$

$$\frac{\partial Q}{\partial x} = -y^2 \qquad \frac{\partial P}{\partial y} = x^2$$



$$\iint_R -(x^2+y^2)\,dA = \text{[illegible]}$$

$R:\ 0 \le r \le 2$

$0 \le \theta \le 2\pi$

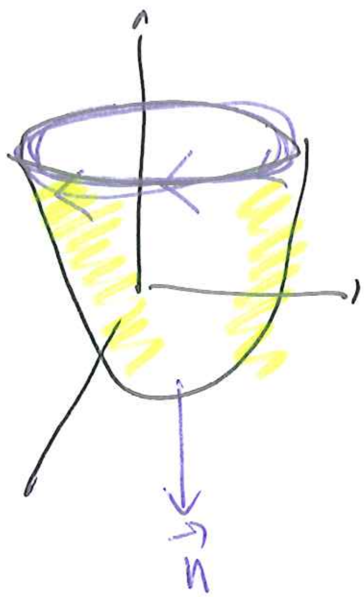
$$\int_0^{2\pi}\int_0^2 (-r^2)\,\boxed{r}\,dr\,d\theta = 2\pi \int_0^2 -r^3\,dr$$

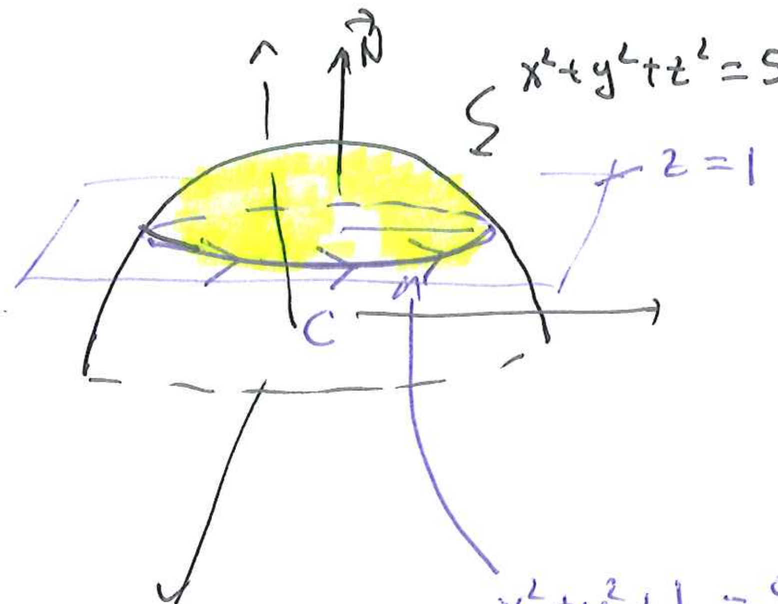
$$= 2\pi(-1)\left[\frac{r^4}{4}\right]_0^2$$

Stokes Thm

$$\oint_C \vec{F} \cdot d\vec{r} =$$

$$\iint_S \text{curl } \vec{F} \cdot d\vec{S}$$





Curve C:

$x(t) = 2\cos t$

$y(t) = 2\sin t$

$z(t) = 1$

$0 \le t \le 2\pi$

$x^2+y^2+1=5$

$x^2+y^2=4$

circle of radius 2

$$\oint_C \vec{F} \cdot d\vec{r} =$$

$$= \int_0^{2\pi} \vec{F}(2\cos t, 2\sin t, 1) \cdot \langle -2\sin t, 2\cos t, 0 \rangle \, dt$$

$$= \int_0^{2\pi} \Big\{ [(2\cos t)^2 \, 2\sin t \cdot 1]\,\hat{i} +$$

$$((2\cos t)\, 1^2]\,\hat{j} +$$

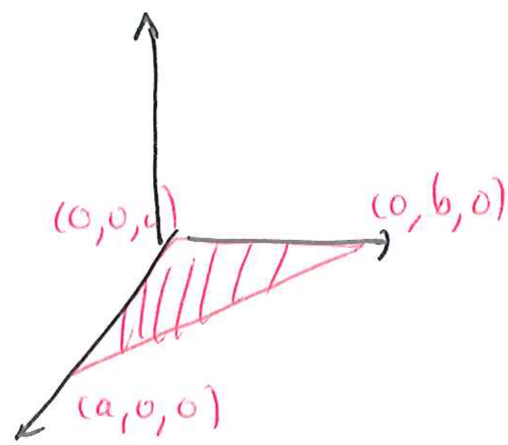
$$[\;\sim\;]\,\hat{k} \Big\} \cdot \langle -2\sin t\,\hat{i} + 2\cos t\,\hat{j} + 0\hat{k} \rangle$$

$$dt$$

$$\text{div } F = \frac{\partial}{\partial x}(z) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(zx)$$

$$= 0 + 1 + x$$

The region:

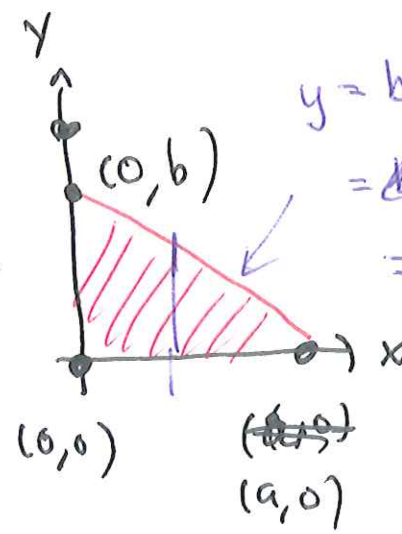


In x, y plane:

$$0 \le x \le a$$

$$0 \le y \le b - \frac{b}{a}x$$

$$0 \le z \le c - \frac{cx}{a} - \frac{cy}{b}$$



$$y = b + mx$$

$$= \cancel{b - \ldots}$$

$$= b - \frac{b}{a}x$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\frac{cx}{a} + \frac{cy}{b} + z = c$$

$$z = c - \frac{cx}{a} - \frac{cy}{b}$$

$$f(x,y) = x^2 + y^2$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

$$\nabla f(x,y) = 2x\,\hat{i} + 2y\,\hat{j}$$

$$f(x,y) = \sqrt{4 - x^2 - y^2}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{4-x^2-y^2}} \cdot (-2x)$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{4-x^2-y^2}} (-2y)$$

At $(x,y) = (1,1)$:

$$\frac{\partial f}{\partial x}(1,1) = \frac{-1}{\sqrt{2}} \qquad \frac{\partial f}{\partial y}(1,1) = -\frac{1}{\sqrt{2}}$$

$$L(x,y) = f(a,b) + \frac{\partial f}{\partial x}(a,b)\,(x-a)$$

$$+ \frac{\partial f}{\partial y}(a,b)\,(y-b)$$

$\underline{\underline{(a,b) = (1,1)}}$:

$$L(x,y) = f(1,1) + -\frac{1}{\sqrt{2}}(x-1) + -\frac{1}{\sqrt{2}}(y-1)$$

$$= \sqrt{2} - \frac{1}{\sqrt{2}}(x-1) - \frac{1}{\sqrt{2}}(y-1)$$

Tangent plane:

$$z = \sqrt{2} - \frac{1}{\sqrt{2}}(x-1) - \frac{1}{\sqrt{2}}(y-1)$$

$$D = \begin{vmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{yx}(a,b) & f_{yy}(a,b) \end{vmatrix}$$

$f(x,y) = x^2 + y^2$

$f_x(x,y) = 2x$

$f_y(x,y) = 2y$

CP : $(0,0)$

$$\begin{vmatrix} \boxed{f_{xx}} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} \boxed{2} & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$f(x,y) = x^2 - y^2$

$$\begin{vmatrix} 2 & 0 \\ 0 & -2 \end{vmatrix} = -4$$

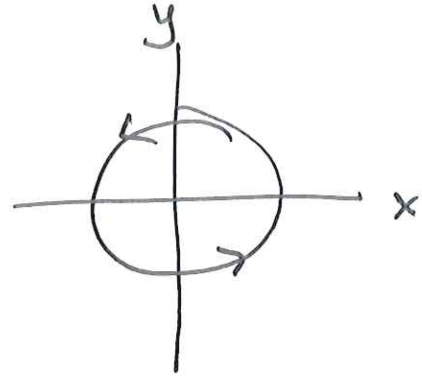
$f(x,y) = x^2 - y^2$

Boundary: circle of radius 1

centered at 0

$x(t) = \cos(t)$

$y(t) = \sin(t)$



$f(x(t), y(t)) = \cos^2 t - \sin^2 t = g(t) \quad 0 \le t \le 2\pi$

$g(0) = 1$

$g(2\pi) = 1$

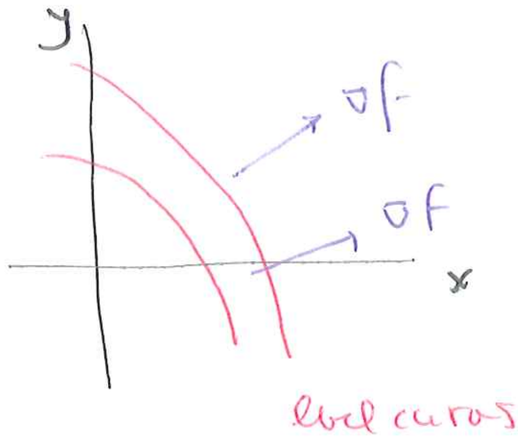
$g'(t) = -2\cos t \sin t - 2\sin t \cos t$

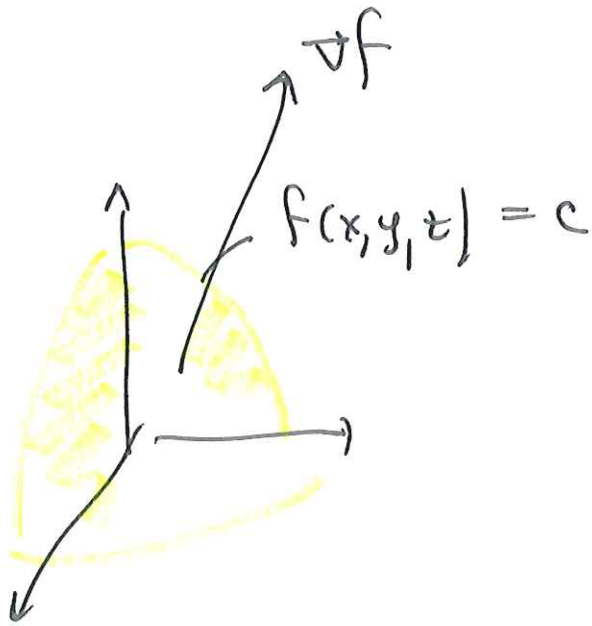
$= -4\cos t \sin t$

$g'(t) = 0$ if $t = \frac{\pi}{2}, \frac{3\pi}{2}, 0, \pi$

$g(\pi) = \cancel{4x} + 1$ $\qquad g\left(\frac{3\pi}{2}\right) = -1$

$g\left(\frac{\pi}{2}\right) = -1$ $\qquad g(0) = 1$





$$\nabla f(x,y,z) = \langle 2x, 8y, 2z \rangle$$

$$\nabla f(2,1,3) = \langle 2, 8, 6 \rangle$$