Math 213 Exam 2

Name: ______________________________ Section: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used other than a one-page "cheat sheet" of notes, formulas, etc., written or typeset on one or both sides of an 8-1/2″ × 11″ sheet of paper. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 6 free-response questions. Please follow these guidelines to receive maximum credit.

- Each question is followed by a space to write your answer. Please write your answer *neatly* in the space provided.
- Show <u>all work</u> to receive full credit on the free response problems. You will be graded on the clarity of your presentation as well as the correctness of your answers.
- Give exact answers, rather than decimal equivalents, unless otherwise instructed (e.g., $\sqrt{2}$, not 1.414).

Question	1	2	3	4	5	6	**Total**
Possible	10	18	18	18	18	18	100
Score							

1. (Limits and Continuity - 10 points) Find:

(a) (6 points) Find $\lim_{(x,y)\to(0,0)} \left(\dfrac{x^3 + y^3}{x^2 y + x y^2} \right)$ or show that it does not exist.

(b) (4 points) Find the set of all points (x, y) for which the function

$$f(x, y) = \ln(9 - x^2 - y^2)$$

is continuous.

2. (Partial Derivatives - 18 points)

   (a) (9 points) Show that the function $u(x, y) = \ln(x^2 + y^2)$ satisfies Laplace's equation

   $$u_{xx} + u_{yy} = 0$$

   by computing u_{xx}, u_{yy} and their sum. It will help to know that

   $$u_x(x, y) = \frac{2x}{x^2 + y^2}, \quad u_y(x, y) = \frac{2y}{x^2 + y^2}.$$

   $u_{xx}(x, y) =$ ______________________

   $u_{yy}(x, y) =$ ______________________

(b) (9 points) Use implicit differentiation to find $\partial z/\partial x$ and $\partial z/\partial y$ if

$$x^2 - y^2 + z^2 - 2z = 4.$$

3. (18 points - Tangent Planes, Linear Approximation)

   (a) (9 points) Find the equation of the tangent plane to the surface $z = x/y^2$ at the point $(-4, 2, -1)$. Express your answer in the form $z = ax + by + c$.

   (b) (9 points) Find the linear approximation to the function

$$f(x, y) = 2 - xy\cos \pi y$$

   at $(1, 2)$ and use it to estimate $f(1.02, 1.97)$

4. (Chain Rule, Directional Derivatives - 18 points)

   (a) (9 points) Suppose that

   $$w = xy^2$$

   and

   $$x = r\cos\theta, y = r\sin\theta.$$

   Use the chain rule to find $\partial w/\partial r$ when $r = 2$, $\theta = \pi/4$. Note that any other solution method will receive no credit.

   (b) (9 points) Find the maximum rate of change of the function $f(x, y) = 4y\sqrt{x}$ at the point $(4, 1)$. Find the direction in which it occurs by computing a unit vector in the direction of greatest change.

5. (Maxima and Minima)

   (a) (9 points) Find the local maximum and minimum values and saddle points for the function

   $$f(x, y) = x^3 - 3x + 3xy^2$$

   For each critical point, write down the Hessian matrix for the critical point, and explain why the critical point is a local maximum, a local minimum, or a saddle.

(b) (9 points) Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$. *Hint*: You can minimize the distance *squared* to make computations easier.

6. (Lagrange Multipliers - 18 points) Find the extreme values (maximum *and* minimum) of the function

$$f(x, y) = xe^y$$

subject to the constraint

$$x^2 + y^2 = 2.$$

Note that a solution by any method other than Lagrange multipliers will receive no credit.