MA 213 Worksheet #25

Sections 16.8 & 16.9

04/23/19

1. *16.8.3,5* Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$

   (a) $\mathbf{F}(x, y, z) = ze^y\mathbf{i} + x\cos(y)\mathbf{j} + xz\sin(y)\mathbf{k}$, S is the hemisphere $x^2 + y^2 + z^2 = 16, y \geq 0$, oriented in the direction of the positive y-axis

   (b) $\mathbf{F}(x, y, z) = xyz\mathbf{i} + xy\mathbf{j} + x^2yz\mathbf{k}$, S consists of the top and the four sides (but not the bottom) of the cube with vertices $(\pm1, \pm1, \pm1)$, oriented outward

2. *16.8.7,10* Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$. In each case C is oriented counterclockwise as viewed from above.

   (a) $\mathbf{F}(x, y, z) = (x + y^2)\mathbf{i} + (y + z^2)\mathbf{j} + (z + x^2)\mathbf{k}$, C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$

   (b) $\mathbf{F}(x, y, z) = 2y\mathbf{i} + xz\mathbf{j} + (x + y)\mathbf{k}$, C is the curve of intersection of the plane $z = y + 2$ and the cylinder $x^2 + y^2 = 1$

3. *16.9.5,7,11* Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

   (a) $\mathbf{F}(x, y, z) = xye^z\mathbf{i} + xy^2z^3\mathbf{j} - ye^z\mathbf{k}$, S is the surface of the box bounded by the coordinate planes and the planes $x = 3$, $y = 2$, and $z = 1$.

   (b) $\mathbf{F}(x, y, z) = 3xy^2\mathbf{i} + xe^z\mathbf{j} + z^3\mathbf{k}$, S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$

   (c) $\mathbf{F}(x, y, z) = (2x^3 + y^3)\mathbf{i} + (y^3 + z^3)\mathbf{j} + 3y^2z\mathbf{k}$, S is the surface of the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the xy-plane