MA 114 Worksheet #16 Solutions: Review for Exam 02

1. List the first five terms of the sequence:

   (a) $a_n = \dfrac{(-1)^n n}{n! + 1}$

   Solution:

   $$a_0 = 0,\ a_1 = -\frac{1}{2},\ a_2 = \frac{2}{3},\ a_3 = -\frac{3}{7},\ a_4 = \frac{4}{25},\ a_5 = -\frac{5}{121}$$

   (b) $a_1 = 6,\ a_{n+1} = \dfrac{a_n}{n}$.

   Solution:

   $$a_1 = 6,\ a_2 = 3,\ a_3 = 1,\ a_4 = \frac{1}{4},\ a_5 = \frac{1}{20}$$

2. Determine whether the sequence converges or diverges. If it converges, find the limit.

   (a) $a_n = 3^n 7^{-n}$

   Solution:

   $$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{3}{7}\right)^n = 0$$

   (b) $a_n = \dfrac{(-1)^{n+1} n}{n + \sqrt{n}}$

   Solution:

   $$\lim_{n\to\infty} a_n \text{ does not exist (oscillates).}$$

   (c) $a_n = \dfrac{\ln n}{\ln 2n}$

   Solution:

   $$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\ln n}{\ln 2 + \ln n} = 1$$

   (d) $a_n = \dfrac{\cos^2 n}{2^n}$

   Solution:

   $$\lim_{n\to\infty} a_n = 0$$

(e) $\{0, 1, 0, 0, 1, 0, 0, 0, 1, \ldots\}$

Solution:

$$\lim_{n\to\infty} a_n \text{ does not exist.}$$

3. Explain what it means to say that $\sum_{n=1}^{\infty} a_n = 2$.

Solution: It means that the sequence of partial sums converges to 2.

4. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^{n-1}}{3^n}$

Solution: This geometric series diverges because the ratio, $\dfrac{4}{3}$, is bigger than one.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{6 \cdot 2^{n-1}}{3^n}$

Solution:

$$\sum_{n=1}^{\infty} \frac{6 \cdot 2^{n-1}}{3^n} = 3\sum_{n=1}^{\infty} \frac{2^n}{3^n} = 3 \cdot \frac{\frac{2}{3}}{1 - \frac{2}{3}} = 6$$

5. Determine whether the given series converges or diverges and state which test you used.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

Solution: Diverges by the Integral Test.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n - 1}$

Solution: Diverges by the Limit Comparison Test with the harmonic series.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{7\sqrt{n}}{5n^{3/2} + 3n - 2}$

Solution: Diverges by the Limit Comparison Test with the harmonic series.

(d) $\sum_{n=1}^{\infty} n! e^{-8n}$

Solution: Diverges by the Ratio Test.

(e) $\sum_{n=1}^{\infty} \left(\frac{\ln n}{5n+7}\right)^n$

Solution: Converges by the n-th Root Test.

(f) $\sum_{n=1}^{\infty} \frac{9^n}{9n}$

Solution: Diverges by the n-th Term Test.

(g) $\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$

Solution: Converges by the Integral Test.

(h) $\sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$

Solution: Diverges by the n-th Term Test.

6. Determine whether the given series is absolutely convergent or conditionally convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{5n+1}$

Solution: Conditionally convergent

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$

Solution: Absolutely convergent

(c) $\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n^2}\right)$

Solution: Divergent

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$

Solution: Divergent

7. Find the radius and interval of convergence of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$

Solution: $RoC = 4$, $IoC = [-4, 4]$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

Solution: $RoC = 1$, $IoC = [-1, 1]$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$

Solution: $RoC = \dfrac{1}{5}$, $IoC = [\frac{3}{5}, 1]$.

8. Find a power series representation for the function and determine its radius of convergence.

(a) $f(x) = \frac{5}{1-4x^2}$

Solution: $f(x) = 5\displaystyle\sum_{n=0}^{\infty} 4^n x^{2n}$ and the radius of convergence is $\dfrac{1}{2}$.

(b) $f(x) = \frac{x^2}{x^4+16}$

Solution: $f(x) = 16\displaystyle\sum_{n=0}^{\infty} \frac{x^{4n+2}}{16^n}$ and the radius of convergence is 2.