MA 114 Worksheet #03: Trig Substitution

1. Compute the following integrals:

   (a) $\displaystyle \int_0^2 \frac{u^3}{\sqrt{16 - u^2}}\, du$

   (b) $\displaystyle \int \frac{1}{x^2\sqrt{25 - x^2}}\, dx$

   (c) $\displaystyle \int \frac{x^2}{\sqrt{9 - x^2}}\, dx$

   (d) $\displaystyle \int \frac{x^3}{\sqrt{4 + x^2}}\, dx$

   (e) $\displaystyle \int \frac{1}{(1 + x)^2}\, dx$

   (f) $\displaystyle \int \frac{\sqrt{1 + x^2}}{x}\, dx$

   (g) $\displaystyle \int \frac{x}{\sqrt{1 + x^2}}\, dx$

   (h) $\displaystyle \int_0^3 \frac{x}{\sqrt{36 - x^2}}\, dx$. Hint: Use the substitution $x = 6u$.

   (i) $\displaystyle \int_0^{1/2} x\sqrt{1 - 4x^2}\, dx$. Hint: Substitute $x = u/2$.

2. Let $r > 0$. Consider the identity

$$\int_0^s \sqrt{r^2 - x^2}\, dx = \frac{1}{2}r^2 \arcsin(s/r) + \frac{1}{2}s\sqrt{r^2 - s^2}$$

   where $0 \le s \le r$.

   (a) Plot the curves $y = \sqrt{r^2 - x^2}$, $x = s$, and $y = \frac{x}{s}\sqrt{r^2 - x^2}$.

   (b) Using part (a), verify the identity geometrically.

   (c) Verify the identity using trigonometric substitution.