## MA 114 Worksheet #25: Calculus with polar coordinates

- 1. Find dy/dx for the following polar curves.
  - (a)  $r = 2\cos\theta + 1$
- (b)  $r = 1/\theta$

- (c)  $r = 2e^{-\theta}$
- 2. In each of the following, compute the slope of the tangent line at the given point. Then sketch the curve and the tangent line.
  - (a)  $r = \sin \theta$  at  $\theta = \pi/3$ .

- (b)  $r = 1/\theta$  at  $\theta = \pi/2$ .
- 3. (a) Give the formula for the area of region bounded by the polar curve  $r = f(\theta)$  from  $\theta = a$  to  $\theta = b$ . Give a geometric explanation of this formula.
  - (b) Give the formula for the length of the polar curve  $r = f(\theta)$  from  $\theta = a$  to  $\theta = b$ .
  - (c) Use these formulas to establish the formulas for the area and circumference of a circle.
- 4. Find the slope of the tangent line to the polar curve  $r = \theta^2$  at  $\theta = \pi$ .
- 5. Find the point(s) where the tangent line to the polar curve  $r = 2 + \sin \theta$  is horizontal.
- 6. Find the area enclosed by one leaf of the curve  $r = \sin 2\theta$ .
- 7. Find the arc length of one leaf of the curve  $r = \sin 2\theta$ .
- 8. Find the area of the region bounded by  $r = \cos \theta$  for  $\theta = 0$  to  $\theta = \pi/4$ .
- 9. Find the area of the region that lies inside both the curves  $r = \sqrt{3} \sin \theta$  and  $r = \cos \theta$ .
- 10. Find the area in the first quadrant that lies inside the curve  $r = 2\cos\theta$  and outside the curve r = 1.
- 11. Find the length of the curve  $r = \theta^2$  for  $0 \le \theta \le 2\pi$ .
- 12. Write down an integral expression for the length of the curve  $r = \sin \theta + \theta$  for  $0 \le \theta \le \pi$  but do not compute the integral.
- 13. Consider the sequence of circles,  $C_n$ , defined by the equations  $x^2 + \left(y + \frac{1}{\sqrt{n}}\right)^2 = \frac{1}{n}$ . Define  $a_n$  as the area of circle  $C_n$  and  $b_n$  as the area between circles  $C_n$  and  $C_{n+1}$ .
  - (a) Sketch the picture of this infinite sequence of circles.
  - (b) Does  $\sum_{n=1}^{\infty} a_n$  converge?
  - (c) Does  $\sum_{n=1}^{\infty} b_n$  converge?
  - (d) Define the circles  $D_n$  by the equations  $x^2 + \left(y + \frac{1}{n}\right)^2 = \frac{1}{n^2}$  with  $d_n$  as the area of  $D_n$ . Does  $\sum_{n=1}^{\infty} d_n$  converge?