Quiz 10

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Find the area enclosed by one loop of $r = 2\sin(2\theta)$. Do not use the calulator to compute the integral.
   Recall: $\sin^2(\theta) = \dfrac{1-\cos(2\theta)}{2}$

Solution: Note that $\sin(2\theta) = 0$ when $2\theta = n\pi$, where n is any integer. The region enclosed by the right loop above the polar axis is swept out by a ray that rotates from $\theta = 0$ to $\theta = \frac{\pi}{2}$. Therefore the area enclosed by one loop is

$$\begin{aligned} A &= \int_0^{\frac{\pi}{2}} \frac{1}{2}(2\sin(2\theta))^2\, d\theta = 2\int_0^{\frac{\pi}{2}} \sin^2(2\theta)\, d\theta \\ &= 2\int_0^{\frac{\pi}{2}} \frac{1-\cos(4\theta)}{2}\, d\theta = \int_0^{\frac{\pi}{2}} (1-\cos(4\theta))\, d\theta \\ &= \left(\theta - \frac{\sin(4\theta)}{4}\right)\Big|_0^{\frac{\pi}{2}} \\ &= \frac{\pi}{2} \end{aligned}$$

2. (2 points) Identify the conic section represented by the equation $9x^2 + 4y^2 = 18x + 27$. Do not use your calculator.

Solution: We complete the square as follows:

$$\begin{aligned} 9x^2 - 18x + 4y^2 &= 27 \\ 9(x^2 - 2x) + 4y^2 &= 27 \\ 9(x^2 - 2x + 1 - 1) + 4y^2 &= 27 \\ 9(x-1)^2 - 9 + 4y^2 &= 27 \\ 9(x-1)^2 + 4y^2 &= 36 \\ \frac{(x-1)^2}{4} + \frac{y^2}{9} &= 1 \quad \text{(divide by 36 on both sides)} \end{aligned}$$

This equations represents an ellipse.