Quiz 5

Name: ______________________ Section and/or TA: ______________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (a) (1 point) State the Integral Test carefully giving the hypotheses and conclusion of the test.

Solution: If f is a continuous, positive, decreasing function on $[1, \infty]$ and if $a_n = f(n)$ for all natural numbers n, then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)\,dx$ converges.

(b) (1 point) **Use the Integral Test** to determinine whether or not the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ converges. You need only show that $f(x) = \frac{x}{x^2+1}$ satisfies a hypothesis about the integral since you may assume that f satisfies the other hypotheses.

Solution: If $u = x^2 + 1$, then

$$\int_1^{\infty} f(x)\,dx = \lim_{R\to\infty} \int_1^R \frac{x}{x^2+1}\,dx = \lim_{R\to\infty} \int_2^R \frac{du}{2u} = \lim_{R\to\infty} \frac{\ln R - \ln 2}{2} = \infty$$

Thus the series $\sum_{n=1}^{\infty} f(n)$ diverges by the Integral Test.

2. (2 points) **Use the Limit Comparison Test** to determinine whether or not the series $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^2-1}$ converges.

Solution: Let $a_n = \dfrac{1}{n}$ and $b_n = \dfrac{n}{n^2-1}$ for natural numbers $n \geq 2$. Then

$$\frac{a_n}{b_n} = \frac{1}{n}\frac{n^2-1}{n} = \frac{n^2-1}{n^2} = 1 - \frac{1}{n^2}$$

for $n \geq 2$ so

$$\lim_{n\to\infty} \frac{a_n}{b_n} = 1.$$

Thus the series $\displaystyle\sum_{n=2}^{\infty} b_n$ diverges since the series $\displaystyle\sum_{n=2}^{\infty} a_n$ is the harmonic series, which diverges.