Quiz 6

Name: ______________________ Section and/or TA: ______________

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Determine if the series $\sum_{n=1}^{\infty} (\sqrt[n]{2} - 1)^n$ converges or diverges.

Solution: Apply the root test:

$$\lim_{n\to\infty} \sqrt[n]{(\sqrt[n]{2} - 1)^n} = \lim_{n\to\infty} \sqrt[n]{2} - 1 = 0 < 1$$

so the series converges absolutely by the root test.

2. (2 points) Find the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 3^n}$. Do not forget to check endpoints of the interval!

Solution: Apply the ratio test:

$$\lim_{n\to\infty} \left| \frac{\frac{x^{n+1}}{(n+1)3^{n+1}}}{\frac{x^n}{n3^n}} \right| = \lim_{n\to\infty} \left| \frac{x^{n+1} n 3^n}{(n+1)3^{n+1} x^n} \right| = \lim_{n\to\infty} \left| \frac{nx}{3(n+1)} \right| = \left| \frac{x}{3} \right| < 1$$

when $-3 < x < 3$, so the radius of convergence is 3.

When $x = -3$ we have $\sum_{n=1}^{\infty} \frac{(-3)^n}{n3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, the alternating harmonic series, which converges conditionally.

When $x = 3$ we have $\sum_{n=1}^{\infty} \frac{3^n}{n3^n} = \sum_{n=1}^{\infty} \frac{1}{n}$, the harmonic series, which diverges.

Hence the interval of convergence is $[-3, 3)$.