

Quiz 7

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Find the average value of $f(x) = 2 \cos(x) - \cos(2x)$ on the interval $[0, \frac{\pi}{2}]$.

$$\begin{aligned}\text{average value} &= \frac{1}{\frac{\pi}{2} - 0} \int_0^{\pi/2} 2 \cos(x) - \cos(2x) \, dx \\ &= \frac{2}{\pi} \left[2 \sin(x) - \frac{1}{2} \sin(2x) \right]_0^{\pi/2} \\ &= \frac{2}{\pi} \left(2 \sin(\pi/2) - \frac{1}{2} \sin(\pi) - 2 \sin(0) + \frac{1}{2} \sin(0) \right) \\ &= \frac{4}{\pi}\end{aligned}$$

2. (2 points) Calculate the volume of the following solid. The base is the region enclosed by $x = 0$, $y = 0$, and $y = \sqrt{1 - x^3}$. The cross sections perpendicular to the x -axis are squares.

Since the cross section is a square, the area $A(x)$ of the cross section is

$$A(x) = \left(\sqrt{1 - x^3} \right)^2 = 1 - x^3.$$

This region extends from $x = 0$ to $x = 1$, so the volume V is

$$V = \int_0^1 A(x) \, dx = \int_0^1 1 - x^3 \, dx = \left[x - \frac{x^4}{4} \right]_0^1 = 1 - \frac{1}{4} = \frac{3}{4}.$$