Quiz 8

Name: _____ Section and/or TA: _____

Answer all questions in a clear and concise manner. Unsupported answers will receive *no credit*.

1. (2 points) Use method of cylindrical shells to find the volume of the solid generated by rotating the region bounded by the curves

$$y = x^3,\ y = 0,\ x = 1,\ x = 2,$$

and rotated about the $y - axis$. Sketch the region to be rotated in the x-y plane.

Solution: A typical shell has radius x, circumference $2\pi x$, and height $f(x) = x^3$. Thus, by the shell method, the volume is

$$V = \int_1^2 2\pi x^4\,dx = \frac{2\pi}{5}x^5\Big|_1^2 = \frac{2\pi(32-1)}{5} = \frac{62\pi}{5}$$

2. (2 points) Find the exact arc length (i.e., do not give a decimal) of the curve $y = 1 + 2x^{3/2}$ for $0 \le x \le \frac{1}{3}$. Sketch the curve.

Solution: Calculate $\frac{dy}{dx} = 3\sqrt{x}$, then

$$\begin{aligned} L &= \int_0^{\frac{1}{3}} \sqrt{1 + (3\sqrt{x})^2}\,dx = \int_0^{\frac{1}{3}} \sqrt{1+9x}\,dx \\ &= \frac{1}{9}\int_1^4 u^{1/2}\,du = \frac{1}{9}\cdot\frac{2}{3}\left[u^{3/2}\right]_1^4 \quad (\text{where } u = 1+9x) \\ &= \frac{2}{27}(4^{3/2} - 1) = \frac{14}{27}. \end{aligned}$$