

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may not receive credit*).

Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.

You are to answer three of the last four questions. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name _____

Section _____

Last four digits of student identification number _____

Question	Score	Total
1		6
2		6
3		6
4		6
5		8
6		8
7		10
8		6
9		6
Q10		12
Q11		12
Q12		12
Q13		12
Free		2
		100

1. Let $f(x) = 1 - x^2$ and $g(x) = 1/x$.

Find $f(g(2))$ and $g(f(2))$.

Find the domain of $f(g(x))$.

$f(g(2)) =$ _____, $g(f(2)) =$ _____, Domain of $f(g(x))$ _____

2. Find the quadratic function $f(x) = ax^2 + bx + c$ which has $f(1) = f(4) = 0$ and $f(2) = 1$.
Give your answer in the form $ax^2 + bx + c$.

$f(x) =$ _____

3. The distance d (in kilometers) a train travels from a station is given as a function of time t (in hours) by the formula: $d(t) = 2t^2 + 5t$, for $t \geq 0$. Find the average velocity of the train between times $t = 2$ and $t = 4$.

Average velocity = _____

4. Suppose that f is continuous and that we know the following limits:

$$\begin{aligned}\lim_{x \rightarrow 0} f(x) &= 4 & \lim_{x \rightarrow 0} g(x) &= 2 \\ \lim_{x \rightarrow 2} f(x) &= 1 & \lim_{x \rightarrow 2} g(x) &= 4 \\ \lim_{x \rightarrow 4} f(x) &= 3 & \lim_{x \rightarrow 4} g(x) &= 0\end{aligned}$$

Compute the following limits and give your reasoning: a) $\lim_{x \rightarrow 4} f(g(x)) =$ _____

b) $\lim_{x \rightarrow 2} (fg)(x) =$ _____

5. Compute the following two limits giving a clear statement of your reasoning:

(a)

$$\lim_{x \rightarrow -1} \frac{x^3 - 1}{x - 1} = \underline{\hspace{2cm}}$$

(b)

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \underline{\hspace{2cm}}$$

6. Are the following functions continuous at the indicated points? Give your reasoning.

(a) $f(x) = \frac{x+1}{x^3+2x-1}$ at $x = 1$.

(b) Is g continuous at $x = 0$ where g is defined by

$$g(x) = \begin{cases} x^2 + 1 & x \geq 0 \\ \sqrt{-x} & x < 0 \end{cases}$$

7. Find the derivatives of the following functions using the differentiation rules. Simplify your answers.

(a) $g(x) = 3x^5 - 2x^3 + 1$

(b) $h(t) = \frac{1+t}{2+t^2}$

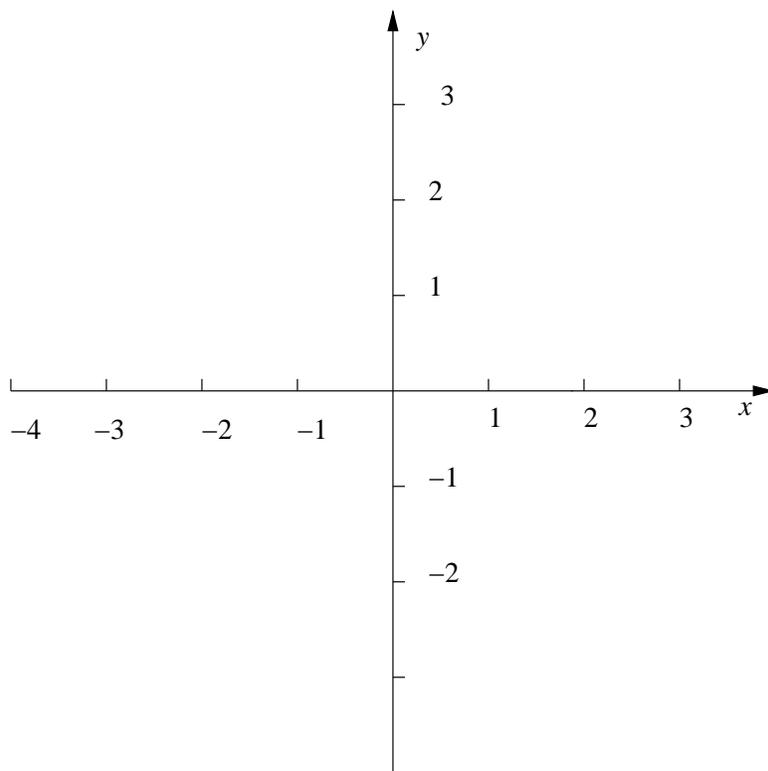
(c) $f(x) = 5x^{-3/5}$

8. Consider the following function:

$$g(x) = \begin{cases} -x & x \leq 0 \\ x^2 & 0 < x \end{cases}$$

(a) Sketch the graph of g .

(b) Is g differentiable at $x = 0$? State your reason why or why not.



9. Find the equation of the tangent line to the graph of the function $f(x) = 2x^2 + 1$ at $x = 1$. Specify the slope and put the equation of the tangent line in the form $y = mx + b$.

Slope $m =$ _____ , Tangent line: $y =$ _____

Answer three of the following four questions. Indicate clearly which question is not to be graded by drawing a line through the question number in the table on the front of the exam.

10. (a) State the principle of mathematical induction. Use complete sentences.
(b) Use the principle of mathematical induction to prove that

$$\sum_{k=1}^n 2k = n^2 + n.$$

11. (a) Suppose the tangent line to the parabola $y = x^2$ at $x = a$ passes through the point $(x, y) = (2, 3)$. Write an equation that a must satisfy.
- (b) Find all solutions to the equation you wrote in part a).
- (c) Find all tangent lines to the parabola $y = x^2$ which pass through the point $(2, 3)$. Put your answer(s) in the form $y = mx + b$.

12. (a) State the definition of the derivative of a function f at a point a . Use complete sentences.
- (b) Use the definition of the derivative to find the derivative of $f(x) = \sqrt{1 + 2x}$
- (c) Give the domain of the derivative, f' .

13. (a) State the intermediate value theorem. Use complete sentences.
- (b) Use the intermediate value theorem to find an interval $[a, b]$ so that the equation $x^5 + 2x^3 - 2x^2 + 4 = 0$ has a solution in the open interval (a, b) .