Calculus I — Russell Brown

Exam 2 — 18 October 2004

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities.
When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit)*.
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.
You are to answer three of the last four questions. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name ____________________

Section __________

Last four digits of student identification number __________

Question	Score	Total
p. 1		12
p. 2		12
p. 3		12
p. 4		12
p. 5		12
Q11		12
Q12		12
Q13		12
Q14		12
Free	4	4
		100

1. Suppose that $\sin(t) = 3/5$ and $\pi/2 < t < 3\pi/2$. Find $\cos(t)$, $\tan(t)$ and $\sin(2t)$.

$\cos(t) =$ __________, $\tan(t) =$ __________, $\sin(2t) =$ __________,

2. Find all values of t in the interval $[0, \pi]$ so that the function $g(t) = 4t + \cos(2t)$ has a horizontal tangent line.

$t =$ ____________________

3. Let $f(x) = \sqrt{13 - x^2}$. Find $f'(2)$.

$f'(2) =$ ______________________

4. Suppose that x and y are related by $x^2 + 2(y-2)^2 = 6$. Find the equation of the tangent line to this curve when $x = 2$ and $y = 1$. Put your answer in the form $y = mx + b$.

$y =$ ______________________

5. A ball is thrown in the air so that its height in meters after t seconds is $h(t) = -5t^2 + 25t$. Find the velocity and acceleration of the ball at time $t = 2$.

velocity ____________________, acceleration ____________________

6. Find a second degree polynomial, $p(t)$, so that $p(1) = 0$, $p'(1) = 0$ and $p''(2) = 4$. Write your answer in the form $at^2 + bt + c$.

$p(t) =$ __________________________

7. Let $f(x) = \sqrt{x}$. Find $L(x)$, the linear approximation to f at $x = 25$. Use this linear approximation to find an approximate value for $\sqrt{24.5}$.

$L(x) =$ ____________________, $\sqrt{24.5} \approx$ ____________________

8. If f is continuous on $[1, 5]$, differentiable on $(1, 5)$, $f(1) = 2$ and $f'(x) \leq 3$, find the largest possible for $f(5)$.

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9. Find the absolute maximum and absolute minimum values of the function $f(x) = x + 2|x - 1|$ for x in the interval $[0, 3]$.

Maximum value ____________, minimum value ____________

10. Let $f(x) = 2x - x^3$. Find an absolute maximum and an absolute minimum for f on the interval $[0, 2]$.

Maximum ________________, minimum ________________

Answer three of the following four questions. Indicate the question that is not to be graded by marking through this question on the front of the exam.

11. (a) Give the values of the limits $\lim_{h\to 0} \dfrac{\sin(h)}{h}$ and $\lim_{h\to 0} \dfrac{1-\cos(h)}{h}$.

(b) Use the limit laws to find the value of the limit $\lim_{h\to 0} \dfrac{1-\cos(h)}{\sin(h)}$

(c) Show how to use the limits in part a) and the addition formula for sin and the definition of the derivative to compute the derivative of the function sin.

12. (a) State Rolle's theorem.

    (b) State the mean value theorem.

    (c) Show how to use Rolle's theorem to prove the mean value theorem.

13. A bicycle wheel of diameter 60 centimeters is rolling along the x-axis. Initially, the wheel is resting at the origin and the center of the wheel is located on the positive y-axis. There is a puddle of purple paint at the origin so the point on the wheel which is located at the origin will be painted purple

    (a) The wheel rolls 10π centimeters to the right. Give the coordinates of center of wheel.

    (b) Does the wheel rotate clockwise or counter-clockwise?

    (c) Give the angle that the wheel has rotated after the wheel rolls 10π centimeters.

    (d) Give the coordinates of the purple point after the wheel rolls 10π centimeters.

    (e) Starting from the origin, the wheel rolls one full turn and the purple paint on the wheel makes a purple dot on the x-axis. How far is this dot from the origin? Assume the wheel rolls without sliding.

14. A very long ladder is 13 meters long and is leaning against a wall that is perpendicular to the ground. The base of the ladder is moving away from the wall at a rate of 2 meters/second.

    (a) If the base of the ladder is 5 meters from the wall, how high is the top of the ladder above the ground?

    (b) Let $y(t)$ be the height of the top of the ladder above the ground. Find the derivative dy/dt when the base of the ladder is 5 meters from the wall.

    (c) In part b) is the top of the ladder rising or falling?