Calculus I
Exam 3

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Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).*

Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.

You are to answer three of the last four questions. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name ______________________

Section ___________

Last four digits of student identification number ___________

| Question | Score | Total |
|---|---|---|
| p. 1 | | 12 |
| p. 2 | | 12 |
| p. 3 | | 12 |
| p. 4 | | 12 |
| p. 5 | | 12 |
| Q11 | | 12 |
| Q12 | | 12 |
| Q13 | | 12 |
| Q14 | | 12 |
| Free | 4 | 4 |
| | | 100 |

1. Suppose that $f$ is a polynomial, $f(1) = 7$, $f'(1) = 0$ and $f''(1) = -7$. Can you determine if $x = 1$ is a local maximum or minimum for $f$? If you cannot, explain why not. If you can, write whether $x = 1$ is a local maximum or minimum and briefly explain your reasoning.

2. Let $f(x) = x^3 + 2x^2$. Is $f$ concave up on the interval $(-1, 0)$? Briefly explain your reasoning.

3. Let

$$f(x) = \frac{x^2 + 3x + 2}{x^2 - x - 2}.$$

Give all horizontal and vertical asymptote(s) to the graph of $f$.

Horizontal asymptote(s) ________________________________

Vertical asymptote(s) ________________________________

4. Suppose that $f$ is a function and the **derivative** of $f$ is

$$f'(x) = \frac{1}{1 + x^2}.$$

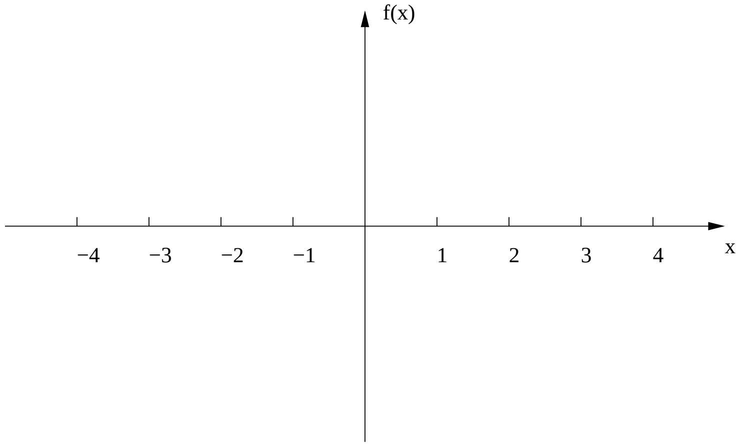
Find all values of $x$ so that $(x, f(x))$ is an inflection point for $f$. Use the inflection points to divide the line $(-\infty, \infty)$ into intervals and determine if $f$ is concave up or concave down on each of the intervals between the inflection points.

Inflection point(s) ________________________________

Interval(s) where $f$ is concave up ________________________________

Interval(s) where $f$ is concave down ________________________________

5. On the axes below, sketch the graph of a function which is defined for $x$ in the set $(-4, 1) \cup (1, 4)$, has a vertical asymptote at $x = 1$, has $f'(x) > 0$ except at $x = 1$, has $f''(x) > 0$ for $x < 1$ and $f''(x) < 0$ for $x > 1$.




6. Suppose that we want to build a closed box with a square base of side length $x$. The volume is to be 64 cubic meters. Write a function $S(x)$ which gives the surface area of this box as a function of $x$, the side length of the base. Give the domain of your function.

$S(x) =$ ____________________, Domain ____________________

7. Consider the equation

$$x^3 - 7 = 0.$$

Use Newton's method beginning with $x_0 = 2$ to find two approximations to a root of this equation. Your answers should be exact or correctly rounded to three decimal places. Show the formula you use to go from $x_n$ to $x_{n+1}$.

$x_1 =$ ____________________, $x_2 =$____________________

8. An object is moving along a vertical line. Let $h$ give height of the object, measured in meters, above a reference point $P$. The object is moving so that $h''(t) = 10$ meters/second$^2$. If $h(1) = 5$ meters and $h'(1) = 20$ meters/second, find $h(t)$ for all $t$.

$h(t) =$__________________________

9. Use a geometric argument of find the value of the integral

$$\int_{-3}^{0} \sqrt{9 - x^2}\, dx.$$

$\int_{-3}^{0} \sqrt{9 - x^2}\, dx =$ ____________

10. Divide the interval $[1, 5]$ into three equal subintervals. Compute the Riemann sum for the integral

$$\int_{1}^{5} \frac{1}{x}\, dx$$

which is determined by these subintervals and which uses the right endpoint of each subinterval as the sample point.

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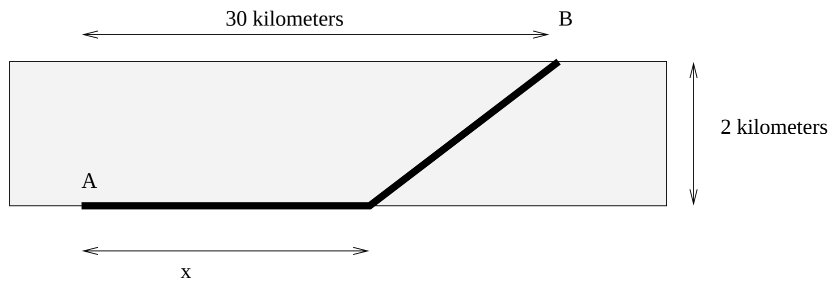
Answer three of the following four questions. Indicate the question that is not to be graded by marking through this question on the front of the exam.

11. Consider an extremely straight canal which is 2 kilometers wide. Suppose that Mary wishes to travel from $A$ to point to a point $B$ which is 30 kilometers downstream from $A$ and on the opposite side of the canal. She will walk $x$ kilometers along the bank from $A$ towards $B$ and then swim diagonally across the canal to point $B$. She will walk at 10 kilometers per hour and she will swim at 5 kilometers per hour.




(a) Find a function $T(x)$ which gives the time it will take Mary to travel from A to B. What is the domain of this function?

(b) Find the value of $x$ which will give the minimum travel time.

(c) Briefly explain how you know you have found a minimum for $T(x)$.

12. For this problem, you may find one or more of the following formulae useful

$$\sum_{k=1}^{N} k = \frac{N(N+1)}{2}, \quad \sum_{k=1}^{N} k^2 = \frac{N(N+1)(2N+1)}{6}.$$

Consider the integral $\int_0^2 x^2\,dx$.

(a) Let the points $P = \{x_0 = 0, x_1, \ldots x_n = 2\}$ divide the interval $[0, 2]$ into $n$ subintervals of equal length. Find a formula for the points $x_k$.

(b) Write out the Riemann sum for $\int_0^2 x^2\,dx$ which uses the points in $P$ to subdivide the interval and the right endpoints of each subinterval as sample points. Let $S_n$ denote this sum.

(c) Find a simple expression for the Riemann sum.

(d) Evaluate $\lim_{n\to\infty} S_n$.

13. (a) State the mean value theorem.

    (b) State the first derivative test for increasing functions.

    (c) Show how to use mean value theorem to prove the first derivative test for increasing functions.

14. Let $f(x) = \frac{2}{3}x + x^{2/3}$.

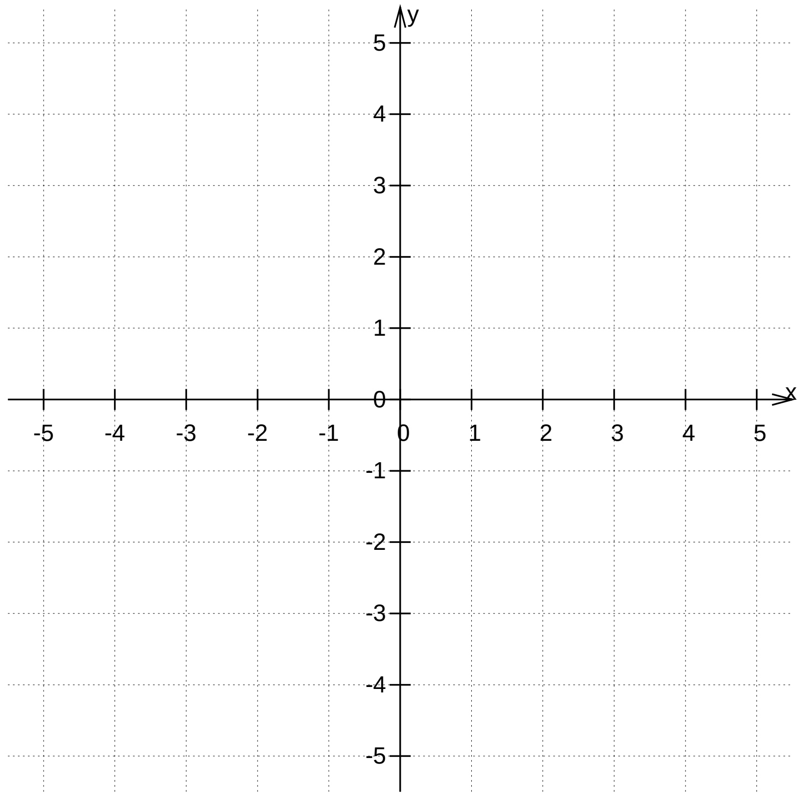
(a) Find the $x$-intercept(s) of the graph of $f$.

(b) Find all critical numbers of $f$.

(c) Give the intervals of increase and decrease for $f$ and find all local extrema for $f$.

(d) Find intervals of concavity for $f$ and find all inflection points.

(e) Sketch the graph of $f$ and label all local extrema and inflection points. (Use the axes on the next page.)


y
x
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1 0 1 2 3 4 5