Calculus I | Russell Brown
Final | 13 December 2005

Answer all of the following questions. Additional sheets are available if necessary. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to 1) check answers when possible, 2) clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers may not receive credit).* 3) exact answers are preferred.
Each question is followed by space to write your answer. Please lay out your solutions neatly in the space below the question. You are not expected to write each solution next to the statement of the question.
You are to answer three of the last four questions. Please indicate which problem is not to be graded by crossing through its number on the table below.

Name ______________________

Section ___________

Last four digits of student identification number ___________

Question	Score	Total
p. 1		12
p. 2		12
p. 3		12
p. 4		12
p. 5		12
Q11		12
Q12		12
Q13		12
Q14		12
Free	4	4
		100

1. Let

$$f(x) = \frac{1}{2+2x^2}.$$

Find the tangent line to the graph of f at $x = -1$. Put your answer in the form $y = mx + b$.

$y =$ ________________________________

2. Find the limit or explain why the limit does not exist.

$$\lim_{x\to 1} \frac{x^2-1}{x^2-2x+1}.$$

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3. Find the tangent line to the curve defined by the equation

$$2x^2 - 3y^3 = 11$$

at the point $(x, y) = (2, -1)$. Put your answer in the form $y = mx + b$.

$y =$ ______________________________

4. Find the linear approximation, $L(x)$, to $f(x) = \sqrt{x}$ at $x = 121$.

Use the linear approximation to find an approximate value for $\sqrt{119}$.

$L(x) =$ ____________________, $\sqrt{119} \approx$ ____________________

5. Suppose that f is function defined for all real numbers x and the derivative of f, $f'(x) = -2x^2$. Find the locations of the absolute minimum value and absolute maximum value of f on the interval $[-1, 2]$.

Minimum at $x =$____________________, Maximum at $x =$ ____________________

6. Let R be the region between $x = 1$, $x = a$, $y = 0$ and $y = 1/x^3$. Find all values of a so that the area of R is 6.

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7. Find the following derivatives.

$$\text{a) } \frac{d}{dx}\int_0^x \frac{1}{1+t^4}dt, \qquad \text{b) } \frac{d}{dx}\int_{x^2}^2 \sin(\sqrt{t})dt.$$

a)______________________________ b)______________________________

8. Find the limit.

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sin\left(\frac{\pi k}{n}\right)$$

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9. If

$$\int_0^2 x + 2f(x)\,dx = 11,$$

find

$$\int_0^2 f(x)\,dx.$$

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10. Evaluate the following integrals

$$a) \int_{-2}^{2} x\sin(x^2)\,dx \qquad b) \int_{\pi/6}^{-\pi/6} \cos(x)\sin^4(x)\,dx.$$

a)____________________ b)____________________

11. (a) State the intermediate value theorem.

(b) Find an interval $[a, b]$ which contains a root of the equation

$$x^3 + 2x = \cos(\pi x).$$

(c) Use the intermediate value theorem to prove that your answer in part b) is correct.

12. (a) State part 1 of the fundamental theorem of calculus.
    (b) State part 2 of the fundamental theorem of calculus.
    (c) Use part 1 of the fundamental theorem to prove part 2 of the fundamental theorem of calculus.

13. A ball is thrown from the ground and 2 seconds after the ball is thrown, the ball is 20 meters above the ground.

    Assume that the acceleration of gravity is 10 meters/second^2.

    (a) Give the height of the ball above the ground as a function of t, the number of seconds after the ball is thrown.
    (b) Give the velocity of the ball at time $t = 0$.
    (c) Give the time that the ball reaches its maximum height.
    (d) Give the maximum height that the ball reaches.

14. Let T be the triangle with vertices $(2, 3)$, $(2, 7)$ and $(4, 7)$. Consider the solid of revolution obtained by rotating T about the line $y = 1$. Use an integral to find the volume of this solid of revolution. You may use the washer method or the shell method. In your answer, you should sketch the triangle, indicate which method you will use, draw a typical strip, write a Riemann sum which represents an approximate value for the volume, take a limit to obtain an integral and evaluate the resulting integral.