M A110 · 9/21/16.

– Study.

R.E.E F #1.

$x(f) = f^2 + 3f$

$y(g) = -g + 3$

$(x+y)(2) = x(2) + y(2)$

$= 2^2 + 3 \cdot 2 + (-2+3)$

$= 4 + 6 + 1$

$= 11$

Operations on functions.

If f, g are two functions, we can define $f+g$ by

$(f+g)(x) = f(x) + g(x)$

f/g , $f-g$

$f \cdot g$

f/g is defined by

$$f/g(x) = \frac{f(x)}{g(x)}.$$

For $x(t) = t^2+3t$

$y(t) = -t+3.$

$(x+y)(t).$

$$= t^2+3t-t+3$$

$$= t^2+2t+3$$

REEF.

$$f \cdot g(x) = x^2-1.$$

$$= f(x)\,g(x)$$

$$= f(x)\,(x+1).$$

$$f(x)\,(x+1) = x^2-1.$$

$$f(x) = \frac{x^2-1}{x+1}$$

$$= \frac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = x-1.$$

Composition of functions—

Start w/ 2 functions, f, g

$(f \circ g)(x) = f(g(x))$

$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x))$

"The composition of g and f."

REEF #3.

$f(x) = \frac{1}{x+1}$

$g(x) = x^2+2$

$(f \circ g)(x) = \frac{1}{(x^2+2)+1}$

$= \frac{1}{x^2+3}$

$g \circ f(x) = \frac{1}{(x+1)^2} + 2$

REEF#4.

$$f(x)=\begin{cases} x, & x \geq 2 \\ x^2, & x < 2. \end{cases}$$

$$g(x)=\begin{cases} |x+2|, & x<-3 \\ 3-2x, & x \geq -3 \end{cases}$$

Find

$$\underbrace{f(f\ldots(f(x))\ldots)}_{2016\ f\text{'s}}$$

2016 f's.

For $f(x) = 1/x$.

$$g(-4) = |-4+2|$$

$$= |-2| = 2$$

$$f(2) = 2.$$

$$f\circ f(x) = 1/f(x) = \frac{1}{1/x} = 1 \cdot \frac{x}{1} = x.$$

$$\underbrace{f\circ f\circ \cdots \circ f}_{n \text{ times}}(x) = \begin{cases} 1/x, & n \text{ odd} \\ x, & n \text{ even.} \end{cases}$$

$$f\circ f\circ f(x) = \frac{1}{f\circ f(x)} = \frac{1}{x}.$$